

WinBUGS : part 1

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Agenda

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 - Data and initial values in WinBUGS
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- Exercice 2 : Comparison of 2 means-unequal variance



WinBUGS

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- ▶ Free software :

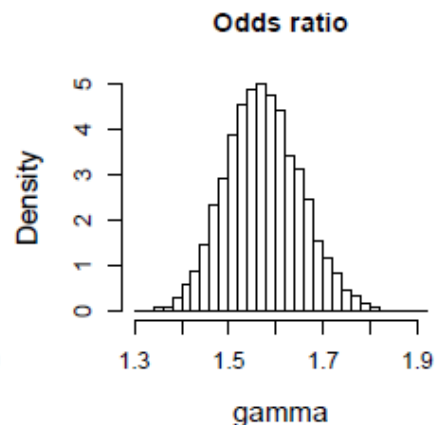
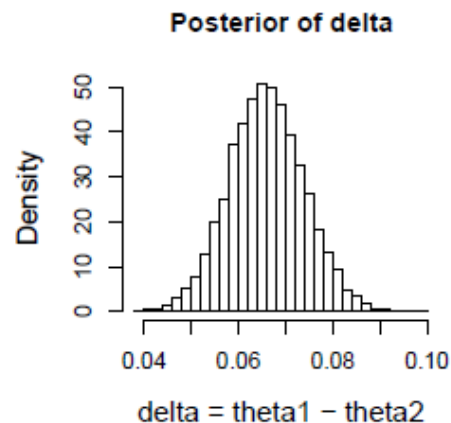
<http://www.mrc-bsu.cam.ac.uk/bugs/winbugs/contents.shtml>

- ▶ WinBUGS implements various MCMC (Markov Chain Monte Carlo) algorithms to generate samples from posterior distributions.
- ▶ WinBUGS: Windows version of the BUGS program (Bayesian analysis using Gibbs Sampling).



```
par(mfrow=c(1,2)) # Split window in 2
M = 10000 ; theta1 = rbeta(M,684,2538) ; theta2 = rbeta(M,1499,8748)
odds1 = theta1/(1-theta1) ; odds2 = theta2/(1-theta2)
delta = theta1-theta2 ; gam = odds1/odds2
hist(delta,breaks=25,freq=F,xlab="delta = theta1 - theta2",
      main="Posterior of delta")
round(mean(delta),3) ; round(quantile(delta,c(.025,.05,.5,.95,.975)),3)
hist(gam,breaks=25,freq=F,xlab="gamma",main="Odds ratio")
round(mean(gam),3) ; round(quantile(gam,c(.025,.05,.5,.95,.975)),3)
```

Comparison of 2 proportions



- Mean and quantiles of δ :

[1]	0.066			
2.5%	5%	50%	95%	97.5%
0.051	0.053	0.066	0.079	0.082

- Mean and quantiles of γ :

[1]	1.574			
2.5%	5%	50%	95%	97.5%
1.422	1.447	1.572	1.712	1.737

- ⊗ Identified posterior distributions are not needed in WinBUGS.



Example description

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- ▶ Assume that it is known from large studies that the mean cholesterol level in children aged 2-14 is 175 mg/dL. It is desirable to determine if there is familial aggregation of cholesterol levels.
- ▶ A group of fathers who had heart attack and elevated cholesterol levels (250 mg/dL) was identified and the cholesterol level of their offspring aged 2-14 was measured.
- ▶ The cholesterol levels of such **100** children have mean **207**mg/dL and standard deviation **30** mg/dL.
- ▶ Is there evidence that the cholesterol in that subpopulation is different from 175 mg/dL? (Rosner 2000, p.184).

Example description

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- ⊙ By experience, it is known that the cholesterol levels are approximately normally distributed.
- ⊙ Therefore, we shall assume that $Y_i \sim N(\mu; \sigma^2)$ ($i = 1, \dots, 100$) where Y_i denotes the cholesterol level of the i th child.
- ⊙ Precision $\tau = 1/\sigma^2$



Example description

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- ▶ **Prior for μ** : $\mu \sim N(\mu_0, \sigma^2_0)$

A large variance σ^2_0 would translate a large uncertainty on the values of μ_0 .

- ▶ **Prior for τ** : $\tau \sim G(a,b)$ with mean a/b and variance a/b^2

A large prior variance would translate uncertainty on the plausible values for τ . A large prior variance can be obtained by taking $a = b = 0.0001$ (say).

Model specification in WinBUGS



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Model specification in WinBUGS

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▶ To specify a Bayesian model, we need:

- **The likelihood:** $Y_i \sim N(\mu; \sigma^2)$ ($i = 1, \dots, 100$)

We use: $\tau = 1/\sigma^2$ (where τ is the precision).

- **Prior distribution of the model parameters:** $\mu; \tau$

- We consider non-informative prior distributions:

$$\mu \sim N(200, 10^4)$$

$$\tau \sim \text{Gamma}(0.0001, 0.0001)$$



Model specification in WinBUGS

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- ⊕ Some common univariate distributions in WinBUGS:

dbin	binomial	$r \sim \text{dbin}(p,n)$
dnorm	normal	$x \sim \text{dnorm}(\mu, \mathbf{\tau})$
dlnorm	log-normal	$x \sim \text{dlnorm}(\mu, \mathbf{\tau})$
dpois	poisson	$r \sim \text{dpois}(\lambda)$
dunif	uniform	$x \sim \text{dunif}(a,b)$
dgamma	gamma	$x \sim \text{dgamma}(a,b)$
dbeta	beta	$x \sim \text{dbeta}(a,b)$

Model specification in WinBUGS

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⊛ Some rules :

- For loop: for (i in 1:n){ }
- Indice: y[i]
- Distribution: $y[i] \sim \text{dnorm}(\mu[i], \tau)$
- Assignment: variance $\leftarrow 1/\tau$ (computed in a deterministic way)



Model specification in WinBUGS

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- **The likelihood:**

```
for (i in 1:n){  
  y[i]~dnorm(mu,tau)}
```

- **Prior distribution of the model parameters:**

```
mu ~ dnorm (200 , 0.0001)
```

```
tau ~ dgamma (0.0001, 0.0001)
```

Model specification in WinBUGS

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```
model{
```

```
#Likelihood
```

```
  for (i in 1:n){
```

```
    y[i]~dnorm(mu,tau)
```

```
  }
```

```
#Prior
```

```
  mu~dnorm(200,0.0001)
```

```
  tau~dgamma(0.0001,0.0001)
```

```
#Computation of the standard deviation
```

```
  sd<- 1/sqrt(tau)
```

```
}
```



Data and initial values in WinBUGS



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Data and initial values

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➤ DATA

➤ Make sure to provide all the data in the model specification.

```
➤ for (i in 1:n){  
  y[i]~dnorm(mu,tau)  
}  
  
#Prior  
mu~dnorm(200,0.0001)  
tau~dgamma(0.0001,0.0001)  
  
#Computation of the standard deviation  
sd<-1/sqrt(tau)
```



Data and initial values

▶ DATA

```
list(y=c(2.51558E+02, 2.25099E+02, 1.66668E+02, 1.68022E+02, 2.19820E+02,
1.77079E+02, 2.49650E+02, 1.87267E+02, 2.02742E+02, 2.20882E+02, 1.98411E
+02, 1.65022E+02, 2.30645E+02, 2.32637E+02, 2.28971E+02, 2.11241E+02,
1.54139E+02, 1.53285E+02, 1.96436E+02, 2.06299E+02, 1.93195E+02, 2.13863E
+02, 1.47117E+02, 2.86469E+02, 1.91467E+02, 2.34219E+02, 1.71060E+02,
1.72452E+02, 1.77047E+02, 2.02564E+02, 2.53186E+02, 2.24139E+02, 1.95245E
+02, 1.50038E+02, 2.21593E+02, 2.05437E+02, 1.76135E+02, 1.92493E+02,
2.09831E+02, 1.77484E+02, 1.99355E+02, 2.21080E+02, 2.15441E+02, 2.76884E
+02, 2.10186E+02, 2.13041E+02, 2.17956E+02, 2.51468E+02, 2.25837E+02,
2.23581E+02, 2.59690E+02, 1.96986E+02, 2.30439E+02, 2.13505E+02, 1.70554E
+02, 2.04125E+02, 2.21069E+02, 2.29274E+02, 2.55735E+02, 2.21125E+02,
2.28745E+02, 2.20769E+02, 2.12672E+02, 2.02715E+02, 2.01250E+02, 2.51791E
+02, 2.17497E+02, 2.26841E+02, 1.95284E+02, 1.95802E+02, 1.89577E+02,
2.36446E+02, 1.79112E+02, 2.12935E+02, 1.93293E+02, 1.84941E+02, 1.48437E
+02, 2.22016E+02, 2.22240E+02, 2.10760E+02, 1.93071E+02, 1.90897E+02,
1.69097E+02, 2.04470E+02, 1.68452E+02, 2.70150E+02, 2.18594E+02, 1.54905E
+02, 2.16406E+02, 2.21260E+02, 2.06473E+02, 1.64567E+02, 2.61956E+02,
1.71259E+02, 2.11077E+02, 2.35920E+02, 2.18069E+02, 1.57796E+02, 2.22255E
+02, 2.05355E+02), n=1.00000E+02)
```


Data and initial values

⊗ Initial values

- ⊗ Make sure to provide initial values for all model parameters.

```
for (i in 1:n){  
  y[i]~dnorm(mu,tau)  
}  
  
#Prior  
mu~dnorm(200,0.0001)  
tau~dgamma(0.0001,0.0001)  
  
#Computation of the standard deviation  
sd<-1/sqrt(tau)
```



Data and initial values

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➤ Initial values

```
list(tau=1.00000E+00, mu=2.00000E+02)
```



Process steps



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Process steps

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- ④ 1- check model
- ④ 2- load data
- ④ 3- compile model
- ④ 4- load initial values
- ④ 5- generate burn-in values
- ④ 6- parameters to be monitored
- ④ 7- perform the sampling to generate samples from the posteriors
- ④ 8- display results



1. Check model

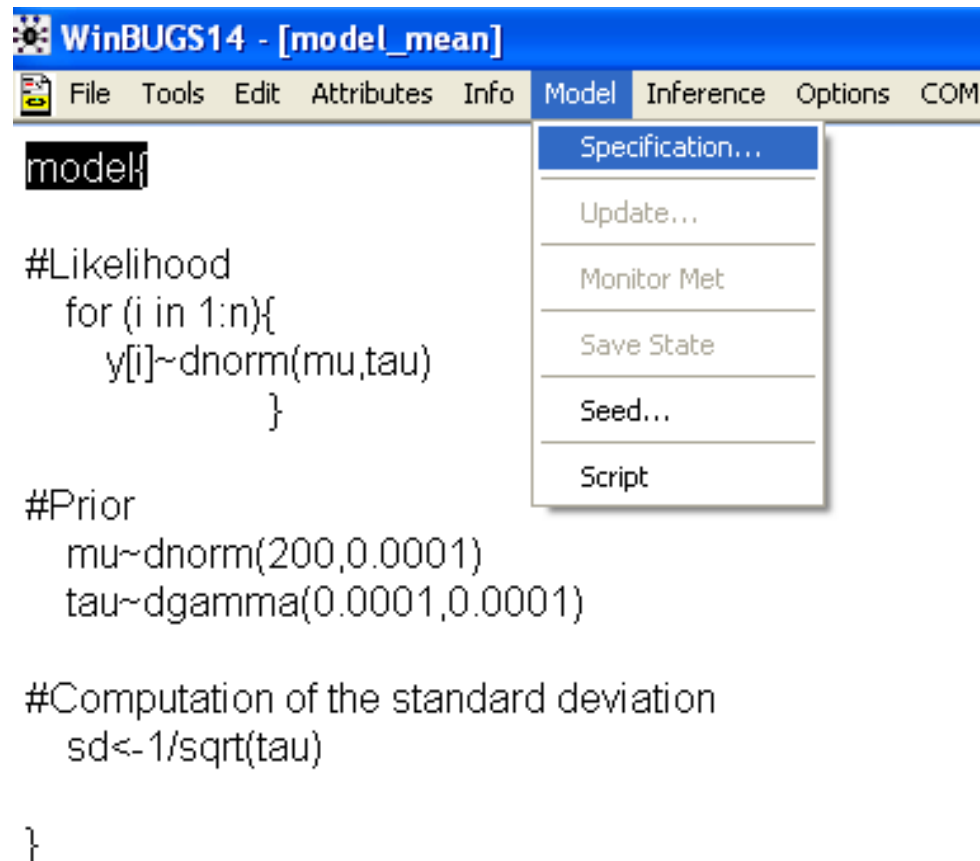


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WinBUGS: step 1

- ⊙ Highlight the word "model"
- ⊙ Go to *Model* -> *Specification* -> *check model*



The screenshot shows the WinBUGS14 application window titled "WinBUGS14 - [model_mean]". The menu bar includes "File", "Tools", "Edit", "Attributes", "Info", "Model", "Inference", "Options", and "COM". The "Model" menu is open, displaying options: "Specification...", "Update...", "Monitor Met", "Save State", "Seed...", and "Script". The "Specification..." option is highlighted. The main text area contains the following WinBUGS script:

```
model{  
  
#Likelihood  
  for (i in 1:n){  
    y[i]~dnorm(mu,tau)  
  }  
  
#Prior  
  mu~dnorm(200,0.0001)  
  tau~dgamma(0.0001,0.0001)  
  
#Computation of the standard deviation  
  sd<-1/sqrt(tau)  
  
}
```



WinBUGS: step 1

The screenshot shows the WinBUGS14 interface. The main window contains the following model code:

```
model{  
  
#Likelihood  
for (i in 1:n){  
  y[i]~dnorm(mu,tau)  
}  
  
#Prior  
mu~dnorm(200,0.0001)  
tau~dgamma(0.0001,0.0001)  
  
#Computation of the standard deviation  
sd<-1/sqrt(tau)  
  
}
```

A 'Specification Tool' dialog box is open, containing the following controls:

- check model (highlighted)
- load data
- compile
- num of chains: 1
- load inits
- for chain: 1
- gen inits

At the bottom of the WinBUGS window, a status bar displays the message: **model is syntactically correct**. This message is also highlighted in a separate box with an arrow pointing to it.

model is syntactically correct

2. Load data



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WinBUGS: step 2

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- ▶ Open your data file (*file -> open*) [.txt file]
- ▶ Highlight the word "list" and click on "load data"

The screenshot shows the WinBUGS14 interface with a data file open. The menu bar includes File, Tools, Edit, Attributes, Info, Model, Inference, Options, COM, Info, Dev, Tools, Controls, Doodle, Map, Obx, and Tut. The main window displays a list of data points in scientific notation, with the word "list" highlighted at the beginning of the first line. Below the main window, the "Specification Tool" dialog box is open, featuring buttons for "check model", "load data", "compile", "load inits", and "gen inits". It also includes input fields for "num of chains" (set to 1) and "for chain" (set to 1).

```
listy=c(2.51558E+02, 2.25099E+02, 1.66668E+02, 1.68022E+02, 2.19820E+02,  
1.77079E+02, 2.49650E+02, 1.87267E+02, 2.02742E+02, 2.20882E+02, 1.98411E+02,  
1.65022E+02, 2.30645E+02, 2.32637E+02, 2.28971E+02, 2.11241E+02, 1.54139E+02,  
1.53285E+02, 1.96436E+02, 2.06299E+02, 1.93195E+02, 2.13863E+02, 1.47117E+02,  
2.86469E+02, 1.91467E+02, 2.34219E+02, 1.71060E+02, 1.72452E+02, 1.77047E+02,  
2.02564E+02, 2.53186E+02, 2.24139E+02, 1.95245E+02, 1.50038E+02, 2.21593E+02,  
2.05437E+02, 1.76135E+02, 1.92493E+02, 2.09831E+02, 1.77484E+02, 1.99355E+02,  
2.21080E+02, 2.15441E+02, 2.76884E+02, 2.10186E+02, 2.13041E+02, 2.17956E+02,  
2.51468E+02, 2.25837E+02, 2.23581E+02, 2.59690E+02, 1.96986E+02, 2.30439E+02,  
2.13505E+02, 1.70554E+02, 2.04125E+02, 2.21069E+02, 2.29274E+02, 2.55735E+02,  
2.21125E+02, 2.28745E+02, 2.20769E+02, 2.12672E+02, 2.02715E+02, 2.01250E+02,  
2.51791E+02, 2.17497E+02, 2.26841E+02, 1.95284E+02, 1.95802E+02, 1.89577E+02,  
2.36446E+02, 1.79112E+02, 2.12935E+02, 1.93293E+02, 1.84941E+02, 1.48437E+02,  
2.22016E+02, 2.22240E+02, 2.10760E+02, 1.93071E+02, 1.90897E+02, 1.69097E+02,  
2.04470E+02, 1.68452E+02, 2.70150E+02, 2.18594E+02, 1.54905E+02, 2.16406E+02,  
2.21260E+02, 2.06473E+02, 1.64567E+02, 2.61956E+02, 1.71259E+02, 2.11077E+02,  
2.35920E+02, 2.18069E+02, 1.57796E+02, 2.22255E+02, 2.05355E+02), n=1.00000E+02)
```

WinBUGS : step 2

The screenshot shows the WinBUGS14 interface with a menu bar (File, Tools, Edit, Attributes, Info, Model, Inference, Options, COM, Info, Dev, Tools, Controls, Doodle, Map, Obx, Tut, SQL, Text, Window, Help) and a main text area containing a list of data points. A 'Specification Tool' dialog box is open, featuring buttons for 'check model', 'load data', 'compile', 'load inits', and 'gen inits', along with input fields for 'num of chains' (set to 1) and 'for chain' (set to 1). A 'data loaded' message box is visible in the lower right, and a status bar at the bottom shows 'data loaded' and 'Allocated Memory: 439776 Bytes'. The Windows taskbar at the bottom includes the Start button and several open applications.

```
list(y=c(2.51558E+02, 2.25099E+02, 1.66668E+02, 1.68022E+02, 2.19820E+02,
1.77079E+02, 2.49650E+02, 1.87267E+02, 2.02742E+02, 2.20882E+02, 1.98411E+02,
1.65022E+02, 2.30645E+02, 2.32637E+02, 2.28971E+02, 2.11241E+02, 1.54139E+02,
1.53285E+02, 1.96436E+02, 2.06299E+02, 1.93195E+02, 2.13863E+02, 1.47117E+02,
2.86469E+02, 1.91467E+02, 2.34219E+02, 1.71060E+02, 1.72452E+02, 1.77047E+02,
2.02564E+02, 2.53186E+02, 2.24139E+02, 1.95245E+02, 1.50038E+02, 2.21593E+02,
2.05437E+02, 1.76135E+02, 1.92493E+02, 2.09831E+02, 1.77484E+02, 1.99355E+02,
2.21080E+02, 2.15441E+02, 2.76884E+02, 2.10186E+02, 2.13041E+02, 2.17956E+02,
2.51468E+02, 2.25837E+02, 2.23581E+02, 2.59690E+02, 1.96986E+02, 2.30439E+02,
2.13505E+02, 1.70554E+02, 2.04125E+02, 2.21069E+02, 2.29274E+02, 2.55735E+02,
2.21125E+02, 2.28745E+02, 2.20769E+02, 2.12672E+02, 2.02715E+02, 2.01250E+02,
2.51791E+02, 2.17497E+02, 2.26841E+02, 1.95284E+02, 1.95802E+02, 1.89577E+02,
2.36446E+02, 1.79112E+02, 2.12935E+02, 1.93293E+02, 1.84941E+02, 1.48437E+02,
2.22016E+02, 2.22240E+02, 2.10760E+02, 1.93071E+02, 1.90897E+02, 1.69097E+02,
2.04470E+02, 1.68452E+02, 2.70150E+02, 2.18594E+02, 1.54905E+02, 2.16406E+02,
2.21260E+02, 2.06473E+02, 1.64567E+02, 2.61956E+02, 1.71259E+02, 2.11077E+02,
2.35920E+02, 2.18069E+02, 1.57796E+02, 2.22255E+02, 2.05355E+02), n=1.00000E+02)
```

3. Compile



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WinBUGS: step 3

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- ▶ Click on "compile"



WinBUGS: step 3

The screenshot displays the WinBUGS14 interface with the following elements:

- Main Window:** Contains a list of data points for a variable 'y'. The list is enclosed in a `list(y=c(...), n=1.00000E+02)` structure. The data points are 100 values in scientific notation, ranging from approximately 1.53285×10^2 to 2.35920×10^2 .
- Specification Tool Dialog:** A smaller window with the following controls:
 - Buttons: `check model`, `load data`, `compile`, `load inits`, `gen inits`.
 - Fields: `num of chains` (set to 1), `for chain` (set to 1).
- Status Bar:** Shows the message `model compiled` and `Allocated Memory: 501'392 Bytes`.
- Taskbar:** Shows the Windows Start button, several open applications (Inbox, Windows Explorer, Internet Explorer, Microsoft Office Word, Microsoft Office PowerPoint), and the WinBUGS application. The system clock shows 2:25 PM.

4. Load initial values



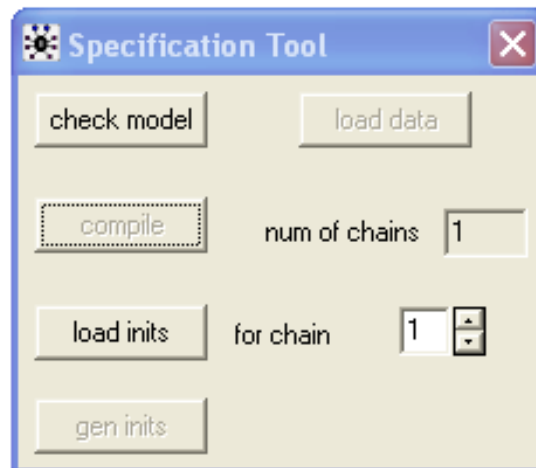
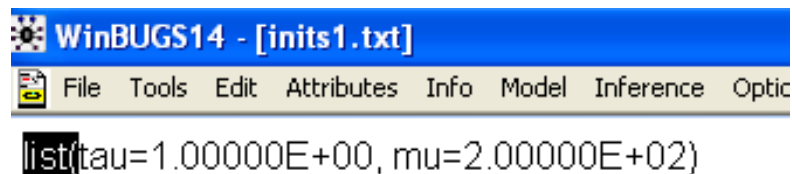
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WinBUGS: step 4

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- ▶ Open your init file (*file -> open*) [.txt file]
- ▶ Highlight the word “list” and click on “load inits”



WinBUGS: step 4

The screenshot displays the WinBUGS14 interface. The main window title is "WinBUGS14 - [inits1.txt]". The menu bar includes File, Tools, Edit, Attributes, Info, Model, Inference, Options, COM, Info, Dev, Tools, Controls, Doodle, Map, Obx, Tut, SQL, Text, Window, and Help. The main text area contains the code: `list(tau=1.00000E+00, mu=2.00000E+02)`. A "Specification Tool" dialog box is open, featuring buttons for "check model", "load data", "compile", "load inits", and "gen inits". It also includes a "num of chains" field set to 1 and a "for chain" field set to 1. A status bar at the bottom of the main window displays "model is initialized" and "Allocated Memory: 573'088 Bytes". A separate box with the text "model is initialized" has an arrow pointing to the status bar text. The Windows taskbar at the bottom shows the Start button and several open applications, with the system clock indicating 2:34 PM.

WinBUGS: step 4

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- ⌚ If you want to run several chains, you have to specify initial values for each chain.
- ⌚ In WinBUGS, give the number of chains after 'load data' and **before 'compile'**



5. Generate burn-in values



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WinBUGS: step 5

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- Go into *Model* -> *Update*
- Specify the burn-in
- Click on update

The screenshot displays the WinBUGS14 software interface. The main window shows a model script with the following code:

```
model{  
  
#Likelihood  
for (i in 1:n){  
  y[i]~dnorm(mu,tau)  
}  
  
#Prior  
mu~dnorm(200,0.0001)  
tau~dgamma(0.0001,0.0001)  
  
#Computation of the standard deviation  
sd<-1/sqrt(tau)  
  
}
```

Two tool windows are open on the right side of the interface:

- Specification Tool:** Contains buttons for "check model", "load data", "compile", "load inits", and "gen inits". It also has a "num of chains" field set to 2 and a "for chain" dropdown menu set to 2.
- Update Tool:** Contains input fields for "updates" (set to 1000) and "refresh" (set to 100). It also has an "update" button, a "thin" field set to 1, and an "iteration" field set to 0. There are checkboxes for "over relax" and "adapting", both of which are currently unchecked.

6. Parameters to monitor



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WinBUGS: step 6

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- Go into *Inference* -> *Samples*

The screenshot displays the WinBUGS14 interface with the following components:

- Model Code:**

```
model{
#Likelihood
for (i in 1:n){
  y[i]~dnorm(mu,tau)
}

#Prior
mu~dnorm(200,0.0001)
tau~dgamma(0.0001,0.0001)

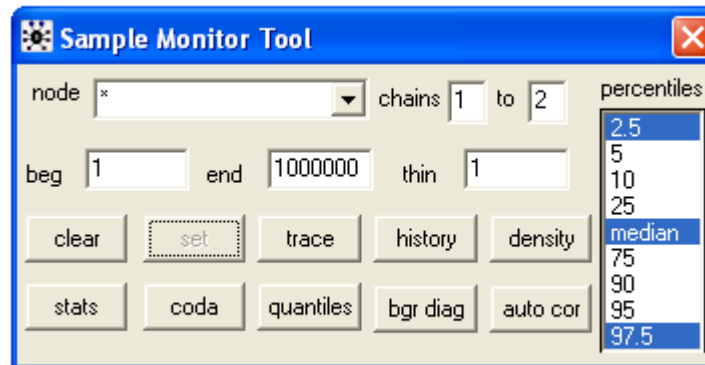
#Computation of the standard deviation
sd<-1/sqrt(tau)
}
```
- Specification Tool:** Contains buttons for 'check model', 'load data', 'compile', 'load inits', and 'gen inits'. It also features a 'num of chains' field set to 2 and a 'for chain' spinner set to 2.
- Update Tool:** Includes 'updates' (1000) and 'refresh' (100) fields, an 'update' button, a 'thin' field (1), and an 'iteration' field (0). It also has checkboxes for 'over relax' and 'adapting'.
- Sample Monitor Tool:** Shows 'node' selection, 'chains' (1 to 2), 'beg' (1), 'end' (1000000), and 'thin' (1) fields. It includes buttons for 'clear', 'set', 'trace', 'history', 'density', 'stats', 'coda', 'quantiles', 'bgr diag', and 'auto cor'. A 'percentiles' list is visible on the right, with values 2.5, 5, 10, 25, median, 75, 90, 95, and 97.5.



WinBUGS: step 6

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- ▶ In "node", write the parameter you want to monitor and click on "set".
- ▶ Repeat for all the parameters you want to monitor: mu, tau and sd.
- ▶ When done, write an asterisk



7. Perform the
sampling to generate
samples from
posteriors



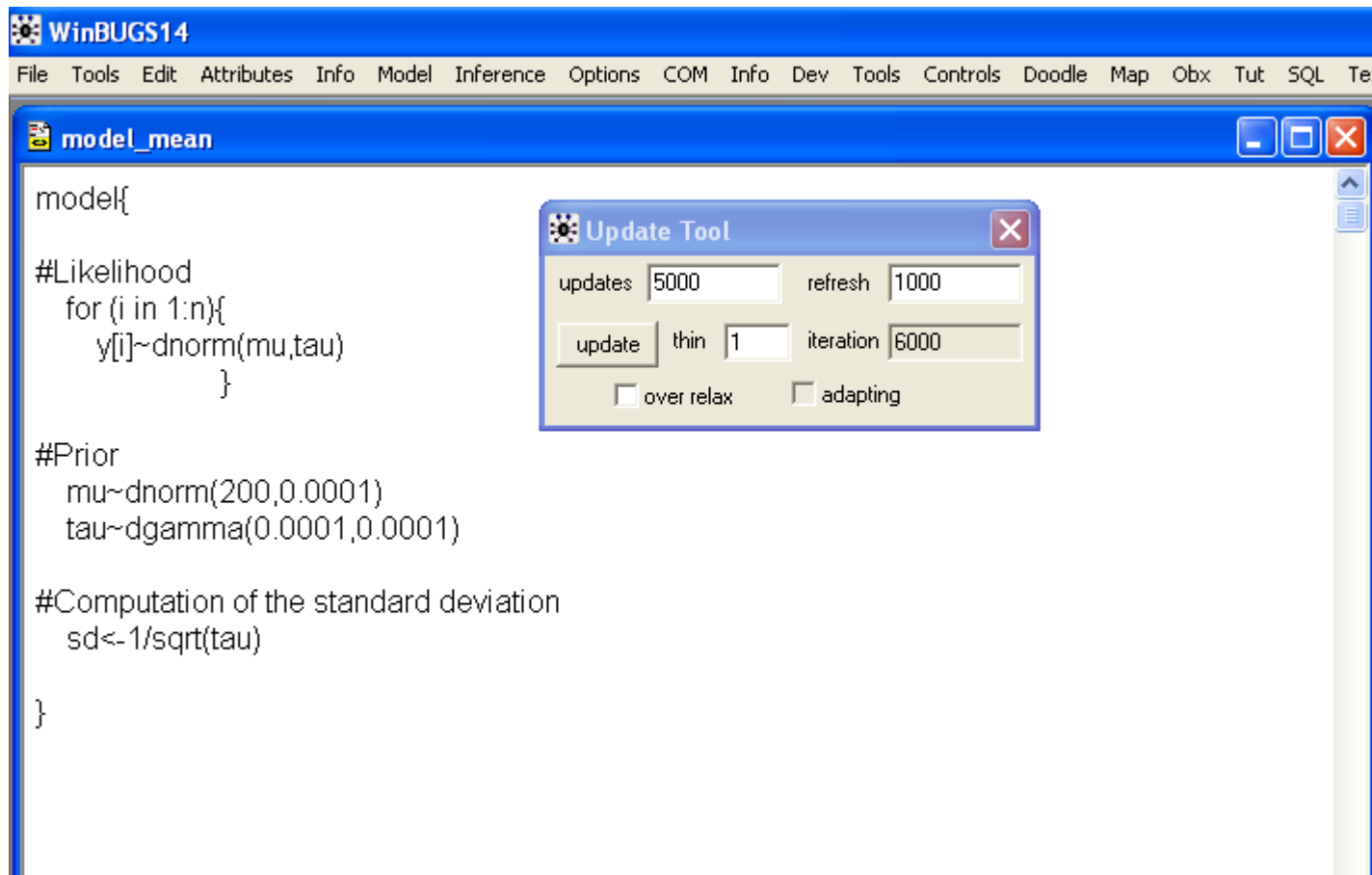
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WinBUGS: step 7

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- ⌚ In the “update tool”, write the number of iterations and click on “update”



The screenshot shows the WinBUGS14 interface. The main window displays a model definition for 'model_mean'. The code is as follows:

```
model{  
#Likelihood  
  for (i in 1:n){  
    y[i]~dnorm(mu,tau)  
  }  
  
#Prior  
  mu~dnorm(200,0.0001)  
  tau~dgamma(0.0001,0.0001)  
  
#Computation of the standard deviation  
  sd<- 1/sqrt(tau)  
}
```

An 'Update Tool' dialog box is open over the code. It contains the following fields and options:

- updates: 5000
- refresh: 1000
- update button
- thin: 1
- iteration: 6000
- over relax
- adapting

8. display the results

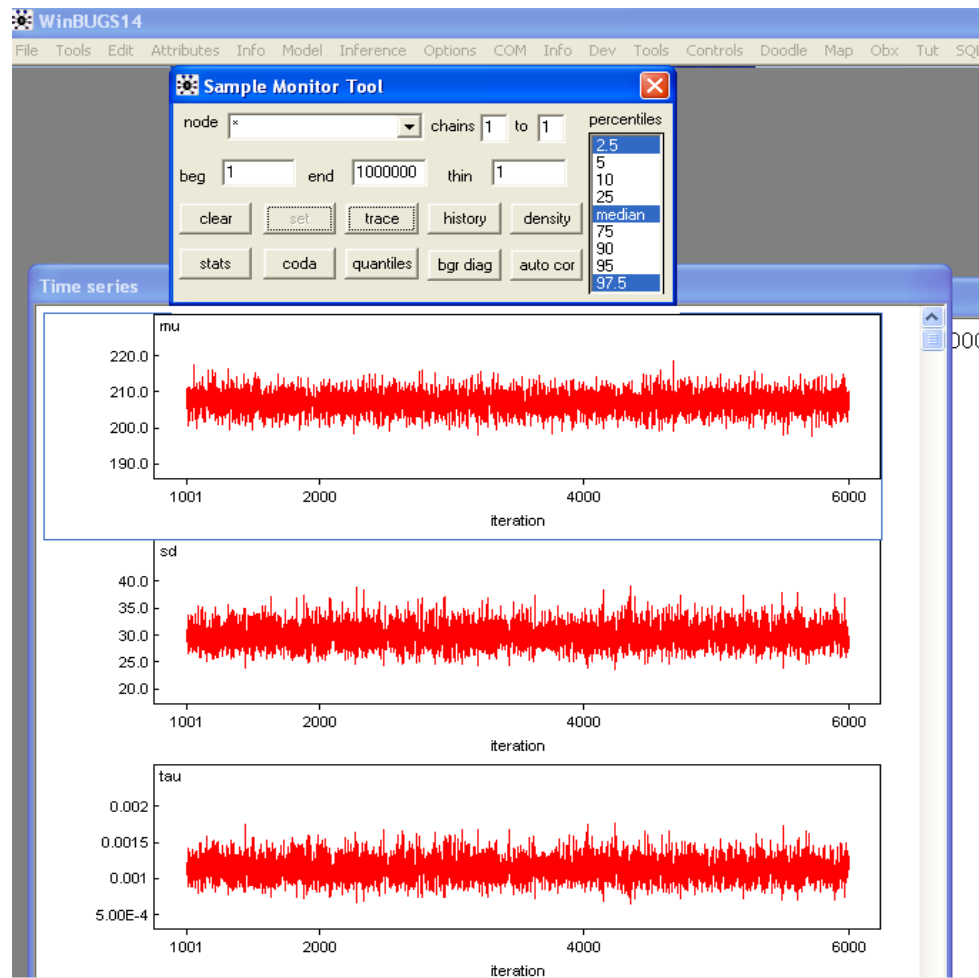


Gabriele,
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WinBUGS: step 8

- In the “sample monitor tool”, click on history to get the traces:



WinBUGS: step 8

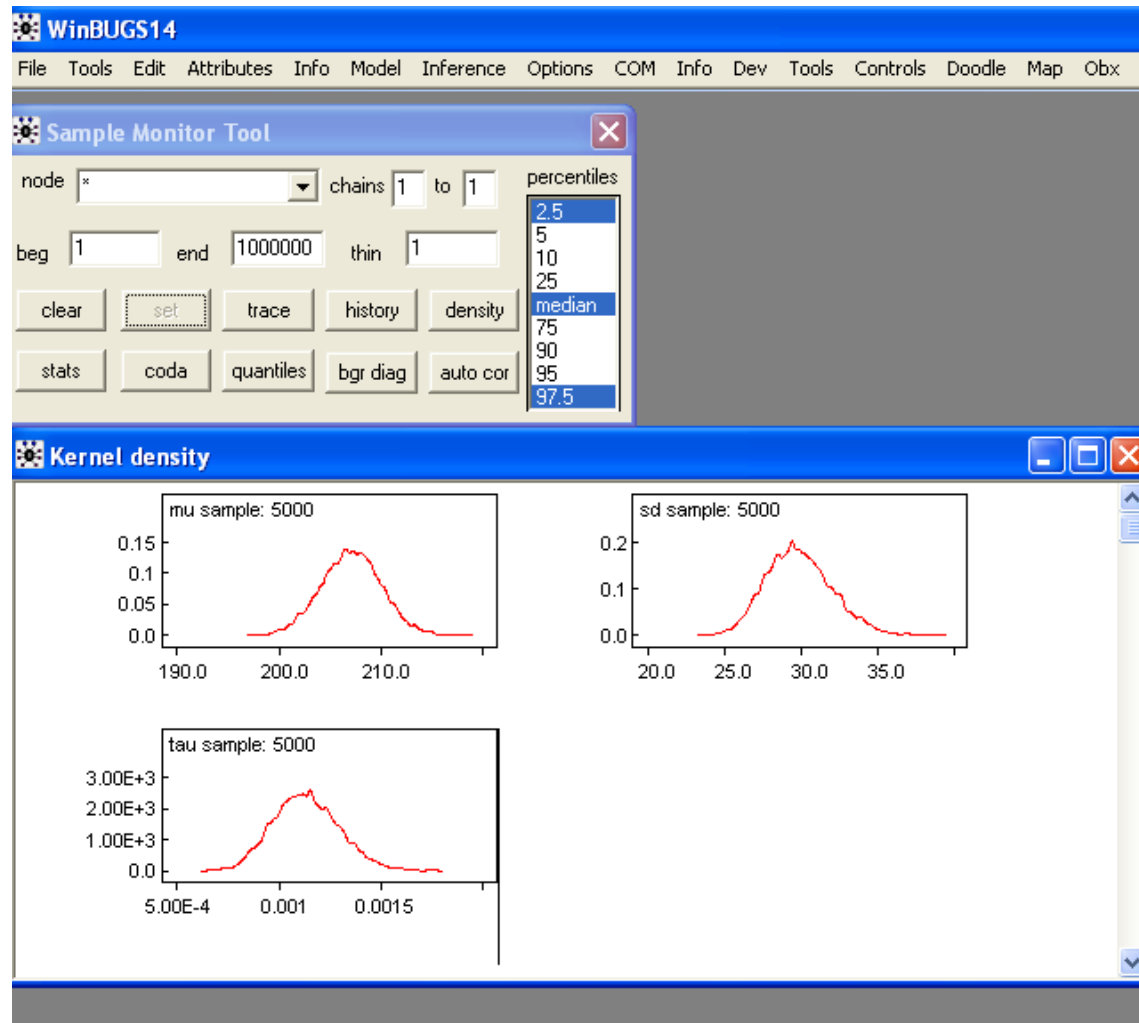
- In the "sample monitor tool", click on "stats":

The screenshot shows the WinBUGS14 interface with the Sample Monitor Tool open. The tool has a menu bar (File, Tools, Edit, Attributes, Info, Model, Inference, Options, COM, Info, Dev, Tools, Controls, Doodle, Map, Obx, Tut, S) and a toolbar with buttons for 'clear', 'set', 'trace', 'history', 'density', 'stats', 'coda', 'quantiles', 'bgr diag', and 'auto cor'. The 'stats' button is highlighted. A 'percentiles' list is open, showing values 2.5, 5, 10, 25, median, 75, 90, 95, and 97.5. Below the tool, a 'Node statistics' window displays a table of data for three nodes: mu, sd, and tau.

node	mean	sd	MC error	2.5%	median	97.5%	start	sample
mu	207.1	2.911	0.04326	201.5	207.1	212.9	1001	5000
sd	29.91	2.152	0.03116	26.1	29.78	34.47	1001	5000
tau	0.001135	1.611E-4	2.405E-6	8.415E-4	0.001128	0.001469	1001	5000

WinBUGS: step 8

- In the "sample monitor tool", click on "density":



WinBUGS: step 8

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- Given that 175 mg/dL is not in the range of plausible values for μ , we conclude that this sub-group of children has a significantly larger mean cholesterol level than the corresponding overall population.



EXERCICE 2:
Comparison of 2 means-
unequal variance



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Exercise 2

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- ⊙ The cholesterol level in 2 groups of children has been measured in a case-control study where the case group consists of $n_1 = 100$ children whose father suffered from cardiovascular diseases.
- ⊙ The mean cholesterol levels of the $n_2 = 74$ control kids is $y_2 = 193.4$ with standard deviation $s_2 = 17.3$. In the case group, one has $y_1 = 207.3$ and $s_1 = 35.6$. (Rosner 2000, p. 287).
- ⊙ The normality hypothesis is reasonable:
$$Y_{1i} \sim N(\mu_1, \sigma^2_1) \quad (i = 1, \dots, n_1)$$
$$Y_{2i} \sim N(\mu_2, \sigma^2_2) \quad (i = 1, \dots, n_2)$$
- ⊙ Estimate the difference between μ_1 and μ_2 : $\delta = \mu_1 - \mu_2$
- ⊙ What do you conclude?

Exercice 2

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- ▶ Non-informative priors :
 - $\mu_1 \sim N(180, 10000)$
 - $\mu_2 \sim N(180, 10000)$
 - $\tau_1 \sim \text{Gamma}(0.0001, 0.0001)$
 - $\tau_2 \sim \text{Gamma}(0.0001, 0.0001)$

with $\tau_1 = 1/\sigma_1^2$ and $\tau_2 = 1/\sigma_2^2$

- ▶ Data : data.txt
- ▶ Initial values : inits1.txt inits2.txt

