



MRC  
Biostatistics  
Unit



UNIVERSITY OF  
CAMBRIDGE

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# **Basket trials – a powerful tool for precision medicine trials**

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# Outline

09:00 – 09:45	Introduction to Basket Trials and Bayesian Inference	Jaki
09:45 – 10:30	Bayesian Borrowing Models	Mozgunov
10:30 – 10:50	Coffee Break	
10:50 – 12:00	Practical	Mozgunov
12:00 – 12:30	How to design a basket trial	Jaki

# Introduction to Basket Trials

Master protocols are a *new* type of study that seeks to answer multiple questions within a single study

- Platform trials
- Basket trials
- Umbrella trials

# Changing paradigms in cancer research

## Categorisation of cancer

*Anatomic location of the primary tumor*

Breast cancer  
Colorectal cancer  
Lung cancer, etc.



*Molecularly defined subtypes*

'Cancer is caused by the alterations in normal genes.'

## Anti-cancer treatment

*Cytotoxic chemotherapy*

'Toxic to cells': disrupt the way cancer cells grow and divide  
Can often affect healthy cells



*Cytostatic drugs*

Targeted therapy  
Immunotherapy

# Groundbreaking approval of immunotherapy

- ★ Keytruda (pembrolizumab), an antibody that attaches to a molecule called PD-1
- ★ To treat unresectable or metastatic solid tumors with a specific biomarker
- ★ First FDA approval of anti-cancer treatment based on biomarkers rather than tumour location
- ★ Read more: [www.fda.gov/newsevents/newsroom/pressannouncements/ucm560167.htm](http://www.fda.gov/newsevents/newsroom/pressannouncements/ucm560167.htm)

# Biomarker-driven designs

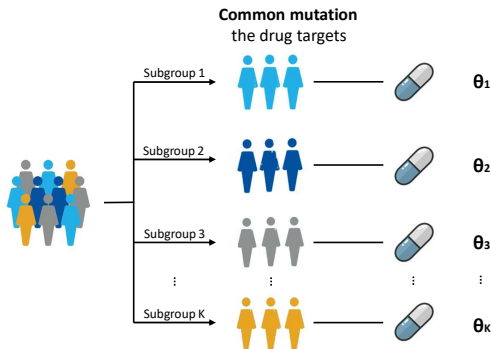
- ★ **Molecular profiling** at the individual patient level became feasible and affordable
- ★ **Biomarker:** e.g., measurable indicator of biological properties or genetic aberration
- ★ More trials are now biomarker-driven
- ★ Choice of the design and analysis relies on the biomarker's nature, e.g., prognostic or predictive
- ★ Well-known types: (adaptive) enrichment designs, master protocols
- ★ **Key objective:** increased efficiency for drug development when target-drug links exist

# Basket trials

**Setting:** Common characteristic (e.g. mutation) present in multiple tumour types.

**Aim:** To develop **targeted therapies**

**Solution** Using biomarker(s) to screen patients and recruit those harbouring a common characteristic (mutation)



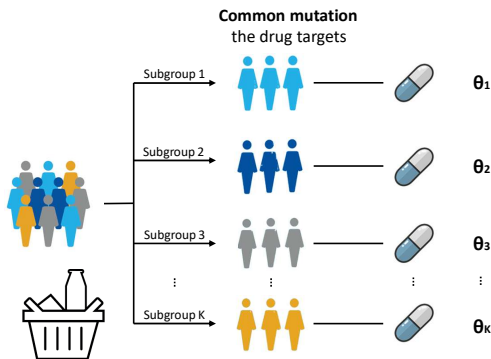


# Basket trials

**Setting:** Common characteristic (e.g. mutation) present in multiple tumour types.

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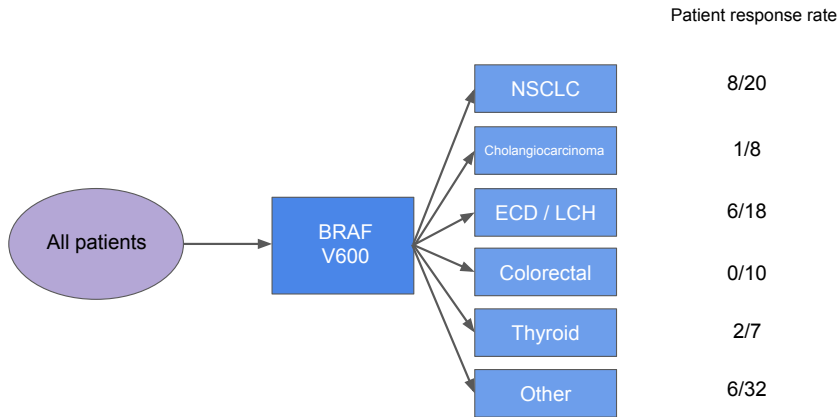
**Solution** Using biomarker(s) to screen patients and recruit those harbouring a common characteristic (mutation)



# Basket trials in oncology – An example

Hyman *et al.* (2015) reported a basket trial, which has been designed to evaluate the efficacy of vemurafenib in BRAF-V600.

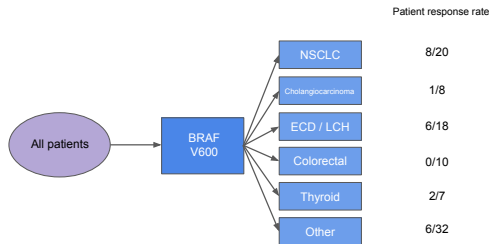
A total of 122 patients with BRAF-V600 mutations were enrolled, of which 95 entered the 6 modules.



# Borrowing of information between modules

With the **common genomic mutation** targeted by the investigational drug, one may expect that

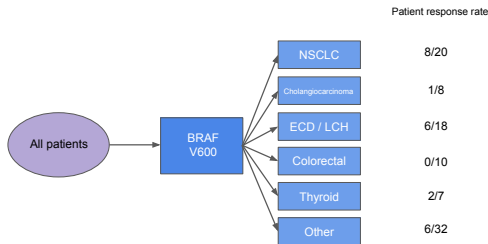
... some patient subgroups will **respond similarly**.



# Borrowing of information between modules

With the **common genomic mutation** targeted by the investigational drug, one may expect that

... some patient subgroups will **respond similarly**.



## Potential analysis strategies:

- Stand-alone analyses
- Complete pooling
- Borrowing of information

# Basket trials outside of oncology

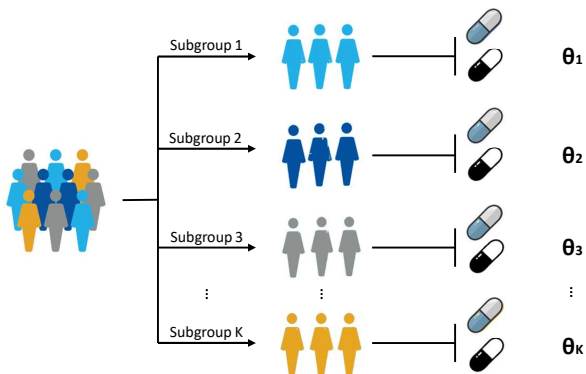
Basket trials evaluate a treatment for multiple disease indications with a **common characteristic**, e.g.

- a genomic biomarker;
- mechanism of drug activity;
- clinical symptom that the treatment targets

There is a **great need** in more efficient study designs utilising this feature of a common characteristic outside of oncology.

- Neurodegenerative (NDD) and neuromuscular diseases: drugs that address aspects of biology or symptoms shared with other NDDs (Cummings et al. 2022)
- Centronuclear myopathies (CNMs): The disorders share a set of common pathologies and phenotypes (Fourage et al. 2021)
- Rare metabolic disorders with causes within the same pathway;
- and many more...

# Randomised controlled basket trials



# Introduction to Bayesian Inference

# Bayesian inference

## Why being Bayesian for Basket trials?

Bayesian inference provides a formal approach for incorporating information from additional sources, e.g.

- expert knowledge;
- historical data;
- **other baskets in the trial;**

Concept of Bayesian inference:



- Our (prior) knowledge about a common characteristic of baskets is incorporated through an *appropriately chosen* prior.



# Example

- Consider a **Phase II study**
- Question: does a new drug provide any benefits in terms of Response Rate ( $\theta_i$ ) in a given disease population (basket)  $i$ ;



- **Binary Outcome:** Response vs No Response;

Responded



Not Responded



# A Bayesian Approach

- Let  $\theta_i$  be the parameter of interest, the probability of response (or a function of it) in basket  $i$ , that is a **random variable** in itself
- $\theta_i$  has a prior distribution  $\pi(\theta_i)$  reflecting our uncertainty / knowledge about it
- Information about nature of  $\theta_i$  comes from the sample  $\mathbf{y} = \{y_1, y_2, \dots, y_n\}$  that has marginal density function  $f(\mathbf{y})$
- The likelihood function  $f(\mathbf{y}|\theta_i)$  is the distribution of  $\mathbf{y}$  conditional on specific values of  $\theta_i$ .
- Bayes' theorem:

$$\pi(\theta_i|\mathbf{y}) = \frac{f(\mathbf{y}|\theta_i)\pi(\theta_i)}{f(\mathbf{y})}$$

## Stand-alone (Independent) Model

**Independent/Stand-alone** analysis is an approach that does not borrow information between baskets and instead conducts stratified analysis for each.

For each basket  $i$ ,

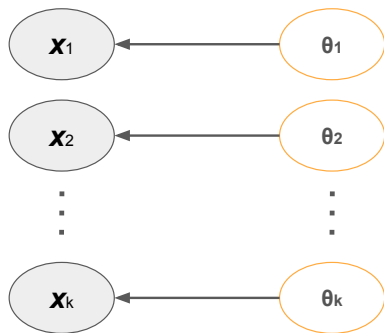
$$\begin{aligned} Y_i &\sim \text{Binomial}(n_i, p_i), \\ \theta_i &= \log\left(\frac{p_i}{1-p_i}\right), \\ \theta_i &\sim \text{N}(m_i, \nu_i), \end{aligned} \tag{1}$$

where  $m_i$  and  $\nu_i$  are parameters of the prior distribution (called **hyperparameters** to distinguish them from the parameters of the sampling space) for basket  $i$ .

# Stand-alone (Independent) Model

Observable data

Subtrial parameters



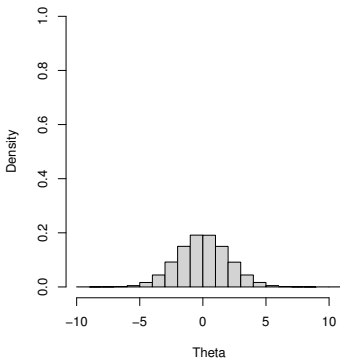
# Prior distribution

Assume a weakly informative prior on parameter  $\theta$  in Model (1)

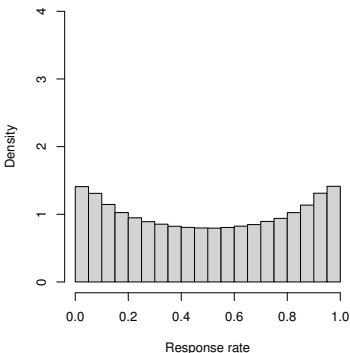
$$\theta \sim N(0.0, 4)$$

Samples from the prior distribution in Model (1) can be obtained

Prior distribution of Theta



Prior distribution of Response Rate



Critical to Bayesian inference are likelihood functions. These

- serve to link the **sampling** space to the **parameter** space.

The probability:

$$f(\mathbf{y}|\theta_i) = L(\theta_i) = \prod_{l=1}^n f(y_l|\theta_i)$$

is a general expression for a likelihood function given *iid* data.

## Example: Bernoulli likelihood

- The observation of 8 responses from 20 patients is a series of Bernoulli trials with  $y = \{0, 1\}$ . For any single trial  $l$  the probability mass function is

$$f(y_l | \theta) = \theta^{y_l} (1 - \theta)^{1 - y_l}$$

- The likelihood function is therefore:

$$L(\theta_i) = f(\mathbf{y} | \theta_i) = \prod_{l=1}^n \theta^{y_l} (1 - \theta)^{1 - y_l} = \theta^{\sum_l y_l} (1 - \theta)^{n - \sum_l y_l}$$

This is the functional form of the *Beta* distribution.

## Note on conjugate prior and MCMC

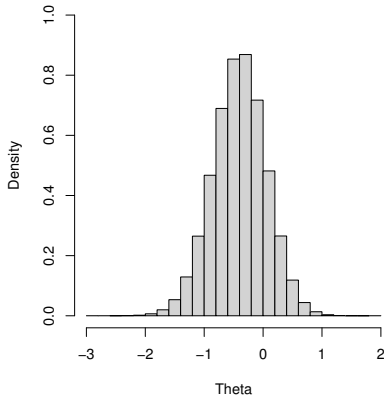
- For the Bernoulli likelihood, if one chooses a prior in the same function form (i.e. the Beta prior distribution), then the posterior will be a Beta distribution again;
- This is called a **conjugate prior**;
- We have chosen a normal (on logit scale) prior to draw a parallel with other (more complicated) models;
- If the prior is not conjugate (as in our case), one can use the **Markov Chain Monte Carlo** (MCMC) to obtain the (approximation) of the posterior distribution of interest;
- The samples from the Markov Chain “approximate” the posterior distribution;



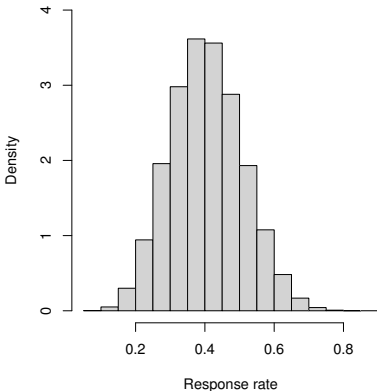
# Posterior distribution

- Assume that 20 patients were treated with 8 responding.
- Prior can be updated into the posterior (again, using MCMC)

Posterior distribution of Theta



Posterior distribution of Response Rate



# Summaries of the posterior distribution

The common summaries of the posterior distribution

- Mean = 0.405
- Median = 0.401
- Standard deviation = 0.104

Posterior means and variances are useful but do not tell us everything.

The posterior distribution can give us various information about the distribution of the parameter.

For example, “*how likely that the response rate is above 25%*”:

$$\mathbb{P}(p > 0.25 | \text{Data, Prior}) = 93.5\%$$

# Credible Interval

A specific region which contains a given area of the posterior density function known as a **Credible Interval**.

**Interpretation:** there is a probability of  $(1 - \alpha) \times 100\%$  that  $\theta$  falls within the region.

- **Highest density region (HDR):** smallest interval for  $\theta$  which contains  $(1 - \alpha) \times 100\%$  of area.
- **Equal-Tailed Interval:** the interval where the probability of being below the interval is as likely as being above it.

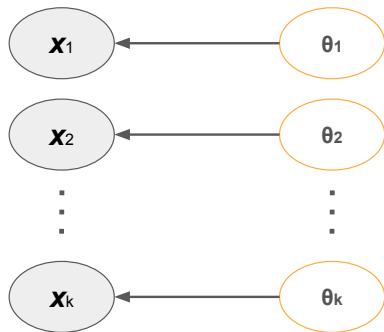
The equal-tailed 95% credible interval in our example is (0.211,0.616)

# Bayesian Borrowing Models

# Borrowing of information

Observable data

Subtrial parameters

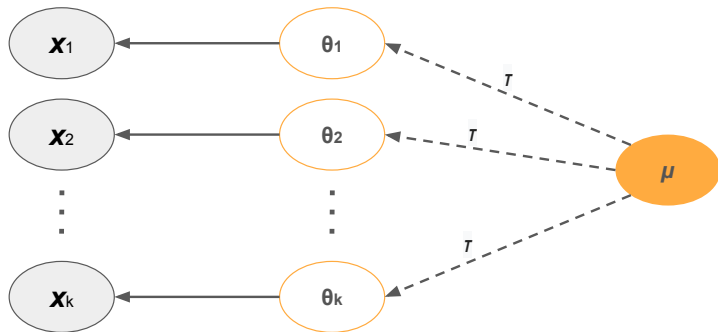


# Borrowing of information

Observable data

Subtrial parameters

Population mean  
(with shrinkage par.)



The random variables  $\theta_1, \theta_2, \dots, \theta_K$  are exchangeable if

$$f_{\theta_1, \theta_2, \dots, \theta_K}(t_1, t_2, \dots, t_K) \stackrel{\text{distr.}}{=} f_{\theta_{\pi_1}, \theta_{\pi_2}, \dots, \theta_{\pi_K}}(t_1, t_2, \dots, t_K),$$

for any permutation  $(\pi_1, \dots, \pi_K)$  of the indices  $\{1, 2, \dots, K\}$ .

It can be shown that

- (1) i.i.d.  $\implies$  exchangeability,
- (2) exchangeability  $\implies$  identically distributed.

A Bayesian hierarchical model (BHM) for binomial data:

$$\begin{aligned} Y_i &\sim \text{Binomial}(n_i, p_i), & i = 1, \dots, k \\ \theta_i | \mu, \sigma &= \log \left( \frac{p_i}{1 - p_i} \right) \sim N(\mu, \sigma^2), \\ \mu &\sim N(\cdot, \cdot), \quad \sigma \sim g(\cdot). \end{aligned} \tag{2}$$

- ★ Hierarchical modelling assumes (similarity) of the  $\theta_i$ s
- Borrowing occurs between all baskets  $\rightarrow$  the estimates of the response rates are shrunk towards the common mean;
- Degree of shrinkage (i.e. borrowing) controlled by the shrinkage (borrowing) parameter,  $\sigma^2$ .
  - ▶  $\sigma^2 = 0 \rightarrow$  complete pooling of data from other baskets;
  - ▶  $\sigma^2 = \infty \rightarrow$  no borrowing.



# Choice of hyper-prior on borrowing parameter

Based on the uncertainty of  $\theta_j$  between different baskets, the following classification of values of  $\sigma$  was proposed

- **Small to moderate** heterogeneity:  $\sigma = 0.125$  to  $\sigma = 0.250$
- **Substantial to large** heterogeneity:  $\sigma = 0.5$  to  $\sigma = 1$

Various choices of hyper-prior  $g(\cdot)$  of  $\sigma$  were proposed with the most common options being

- **Inverse-Gamma**

Used in the original BHM proposal but was found to lead to a poor behaviour when  $\sigma^2$  is close to 0 (Cunanan et al. 2019)

- **Half-Cauchy** (with a moderately large scale was suggested instead), e.g. Half-Cauchy(0, 25) (Gelman 2006)
- **Half-Normal** with a prior standard deviation of  $s = 0.5$  [95% interval on  $\sigma$  is (0.02, 1.12)] or  $s = 1$  [95% interval on  $\sigma$  is (0.03, 2.24)] (Neuenschwander et al. 2016)

## Back to the example - I

Now instead of analysing the first basket (8/20) alone, we conduct the analysis together with the second basket (6/18).

We use the BHM with Half-normal prior on  $\sigma^2$ .

The summary characteristics for the first basket:

	Independent	BHM ( $s = 0.5$ )	BHM ( $s = 1.0$ )
Mean	0.405	0.380	0.385
Median	0.402	0.377	0.381
SD	0.104	0.087	0.091
95% CI	(0.211,0.616)	(0.221,0.559)	(0.218,0.574)
Length of CI	0.404	0.338	0.357

## Back to the example - II

Assume that we were less lucky with the second basket and actually the number of responses was 0/18.

We again use the BHM with Half-normal prior on  $\sigma^2$ .

The summary characteristics for the first basket:

	Independent	BHM ( $s = 0.5$ )	BHM ( $s = 1.0$ )
Mean	0.405	0.321	0.362
Median	0.402	0.314	0.358
SD	0.104	0.102	0.105
95% CI	(0.211,0.616)	(0.145,0.536)	(0.172,0.579)
Length of CI	0.404	0.391	0.407

- If the baskets are **truly homogeneous**, the gains are higher if smaller variance on the borrowing parameter is assumed
- However, if there is a **heterogeneous basket**, then borrowing can lead to too much shrinkage of the estimates and even increased variance of the estimate
- The choice of the distribution of the borrowing parameters  $\sigma^2$  should be based on the trade-off between these two cases.

The calibrated BHM, CBHM, has the same form as the BHM, but rather than placing a prior on  $\sigma$ , it defines a measure of homogeneity:

$$\sigma^2 = \exp\{a + b \log(T)\}$$

where  $T$  is the chi-squared test statistic for homogeneity:

$$T = \sum_{i=1}^k \frac{(O_{0i} - E_{0i})^2}{E_{0i}} + \sum_{i=1}^k \frac{(O_{1i} - E_{1i})^2}{E_{1i}}, \quad (3)$$

where  $O_{0i}$  and  $O_{1i}$  are the observed failures/responses, while  $E_{0i}$  and  $E_{1i}$  are the expected failures/responses.

The parameters  $a$  and  $b$  are tuned via simulations to ensure

- borrowing when all baskets have a homogeneous response
- treat baskets as independent otherwise

- A benefit of the tuning procedure is the increased certainty in estimates produced by the CBHM in comparison to the BHM in the case where all baskets are homogeneous.
- However, the method takes on a '**strong**' definition of **heterogeneity**: if the response rate in one basket is heterogeneous, then all baskets are deemed heterogeneous, and as a result no borrowing occurs.
- Calibration procedure depends on the chosen scenarios and the sample sizes (!) assumed for these scenarios

# Relaxing exchangeability assumption – EXNEX

Neuenschwander et al. 2016

A Bayesian hierarchical model (BHM) assumes that the basket are exchangeable ... with probability of 1 (!).

The full exchangeability assumption is often violated. To tackle this, an EXNEX model was proposed

1. EX (exchangeable component): with prior probability  $w_i$ , basket  $i$  is exchangeable and BHM is applied.
2. NEX (nonexchangeable component): with prior probability  $1 - w_i$ ,  $\theta_i$  is nonexchangeable with any other basket and analysed independently.

$$\begin{aligned} Y_i &\sim \text{Binomial}(n_i, p_i), & M_{1i} &\sim \text{N}(\mu, \sigma^2), & (\text{EX}) \\ \theta_i &= \log\left(\frac{p_i}{1 - p_i}\right), & \mu &\sim \text{N}(\cdot, \cdot), \\ \theta_i &= \delta_i M_{1i} + (1 - \delta_i) M_{2i}, & \sigma &\sim g(\cdot), \\ \delta_i &\sim \text{Bernoulli}(w_i), & M_{2i} &\sim \text{N}(m_i, \nu_i). & (\text{NEX}) \quad (4) \end{aligned}$$

## Back to the example - I

We again analyse the first basket (8/20) together with the second basket (6/18).

We use the EXNEX with Half-normal prior on  $\sigma^2$  with  $s = 0.5$ .

The summary characteristics for the first basket:

	Independent	BHM ( $s = 0.5$ )	EXNEX ( $w = 0.5$ )
Mean	0.405	0.380	0.395
Median	0.402	0.377	0.390
SD	0.104	0.087	0.097
95% CI	(0.211,0.616)	(0.221,0.559)	(0.216,0.597)
Length of CI	0.404	0.338	0.381



## Back to the example - II

Again, assume that we were less lucky with the second basket and actually the number of responses was 0/18.

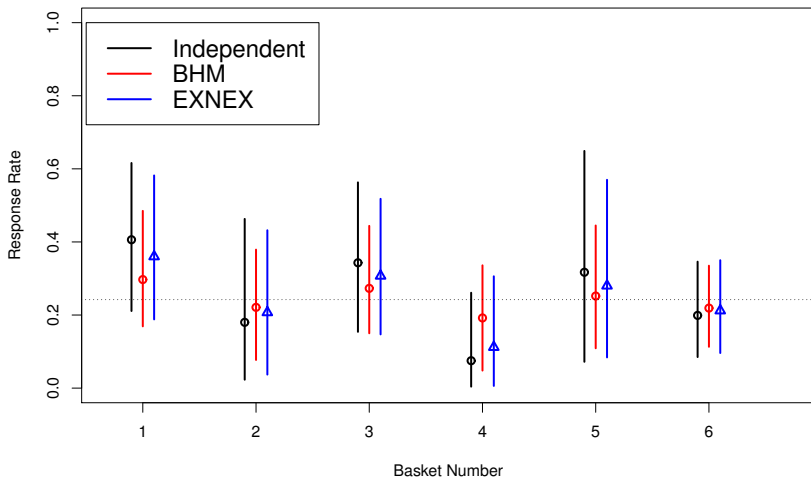
We use EXNEX with Half-Normal prior on  $\sigma^2$  with  $s = 0.5$ .

The summary characteristics for the first basket:

	Independent	BHM ( $s = 0.5$ )	EXNEX ( $w = 0.5$ )
Mean	0.405	0.321	0.402
Median	0.402	0.314	0.399
SD	0.104	0.102	0.105
95% CI	(0.211,0.616)	(0.145,0.536)	(0.204,0.614)
Length of CI	0.404	0.391	0.409

- EXNEX provides more flexibility: information is borrowed only between baskets assigned to the EX component but not from those in the NEX component;
- Weight  $w_i$  is the **prior** probability reflecting how likely we believe (a-priori) that the baskets are exchangeable
- It is uncommon to have strong prior information on the probability of exchangeability, so it is suggested to fix  $\pi_i = 0.5$
- This prior probability is updated to a some degree based on the homogeneity of the data but...
- The update might not be sensitive enough to the heterogeneity/homogeneity of responses in the typically small sample sizes

# Comparing different analysis models: VE-Basket

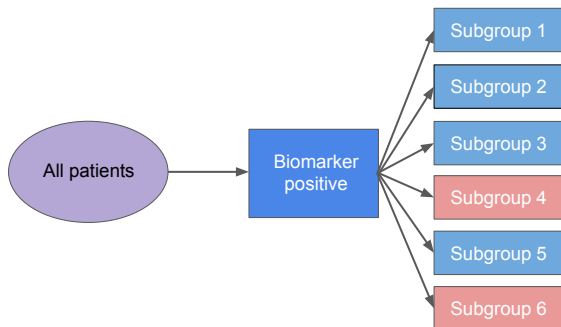


# Can we do anything better?

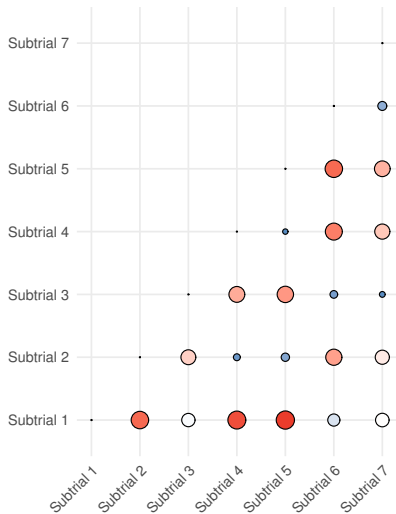
Some modules may display more similar treatment effects between themselves than with others.

Pertinent questions to the EXNEX approach:

- ★ How many EX distributions are needed?
- ★ What is an 'extreme subgroup'?



# Pairwise similarity



The  $mEXNEX_c$  model is a two-step procedure

- Remove baskets with a clearly heterogeneous response rate

$$\min_{i'} \{ |\hat{\rho}_i - \hat{\rho}_{i'}| \} > c, \quad i \neq i',$$

Treat such a basket as independent and its mixture weight in the EXNEX model is set to 0.

- Find Hellinger pair-wise distance between posteriors (under independence) of basket  $i$  and  $i'$ :  $h_{i,i'}$ . Compute the weight as average of these distances

$$w_i = \sum_{i'} \frac{1 - h_{i,i'}}{\# \text{ of baskets} - 1} \quad \text{for } i, i' \in S, \quad i \neq i'.$$

- Use these weights in the EXNEX



- ★ Construct a *complete* model space with all possible reclassification of subgroups with identical  $p_i$
- ★ Each model  $\mathcal{M}_\ell$  corresponds to a distinct model stipulation

Define  $\mathcal{M}_j$  as model  $j$  representing a permutation of basket allocation to the EX group or NEX group.

In the EX group, results are pooled and baskets have one shared response rate.

A weakly informative Beta prior is placed on the response rates, while a prior on each model,  $f(\mathcal{M}_j)$ , is also required. The posteriors  $f(p_i|\mathcal{M}_j)$  and  $f(\mathcal{M}_j|D)$  are computed.

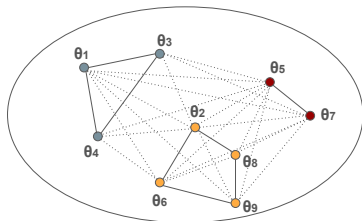
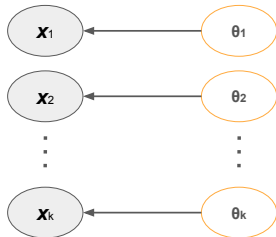
BMA procedure to obtain a summary statistic for basket  $k$ . For example, for the probability of being above 0.25

$$\mathbb{P}(p_i > 0.25 | \text{Data, Prior}) = \sum_j \mathbb{P}(p_i > 0.25 | \mathcal{M}_j, D) f(\mathcal{M}_j | D)$$



Observable data

Subtrial parameters



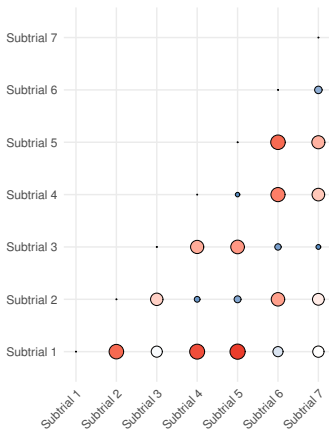
**Aim:** to estimate the treatment effects,  $\theta_i$ , using entire trial data.

# Robust borrowing of information

$$\begin{aligned}\theta_i \mid \theta_q, \nu_{qi} &\sim N(\theta_q, \nu_{qi}^{-1}), \forall i = 1, \dots, k \\ \nu_{qi} &\sim w_{qi} \text{Gamma}(a_1, b_1) + \\ &\quad (1 - w_{qi}) \text{Gamma}(a_2, b_2), \text{ with } q \neq i\end{aligned}$$

- $\text{Gamma}(a_1, b_1)$  correspond to substantial borrowing
- $\text{Gamma}(a_2, b_2)$  correspond to limited borrowing
- Setting  $w_{qi} \rightarrow 0$  means strong borrowing and 1 means no borrowing.

# Robust borrowing of information



- $w_{qi}$  measures dissimilarity
- e.g. can be based on Hellinger Distance

$$w_{qi} = \left[ 1 - \sqrt{\frac{2\sigma_q\sigma_i}{\sigma_q^2 + \sigma_i^2}} \exp\left(-\frac{(\mu_q - \mu_i)^2}{4(\sigma_q^2 + \sigma_i^2)}\right) \right]^{1/2} \quad \text{for } Y_i \sim N(\mu_i, \sigma_i^2)$$

## Obtaining a collective prior

Given all pairwise commensurate priors  $\pi(\theta_i|\mathbf{y}_q)$ ,  $\forall i \neq q$ , we then synthesise  $K - 1$  commensurate predictive priors.

The synthesis weights are

$$d_{qi} = \frac{\exp(-w_{qi}^2/r_0)}{\sum_q \exp(-w_{qi}^2/r_0)}$$

where  $\sum_q d_{qi} = 1$  and  $r_0$  is the parameters that governs how much influence the Hellinger distance has on the weight.

The resulting collective commensurate predictive prior

$$\theta_i|\mathbf{y}_{-i} \sim \mathcal{N} \left( \sum_{q \neq i} d_{qi} \lambda_q, \sum_{q \neq i} d_{qi}^2 \xi_{qi}^2 \right)$$

where  $\lambda_q$  is the mean of  $\theta_q|\mathbf{y}_q$  and  $\xi_{qi}^2$  is the variance of  $\theta_i|\mathbf{y}_q$

# How to design a basket trial

# Decision-making under Bayesian inference

- We have covered various analysis methods, but how one can plan for such a trial?
  - ▶ What would be the type I error? Power?  
If (some of) the baskets are not homogeneous, one can expect an inflation of type I or/and decrease in power
  - ▶ What are the properties of the estimation?
- A conventional approach is to **fix the sample size** (assuming no borrowing or just based on feasibility) and then conduct simulation studies to evaluate the operating characteristics
- An alternative would be to actually **plan for the study** (i.e. for the sample size) **under the assumption of borrowing**
- We will cover both approaches

- Decision are made based on the **posterior probabilities for the treatment effect** given the clinical trial data;

For example, in a single-arm trial, one is claiming **benefit** if

$$\text{Prob}[\theta \geq \rho_0 \mid \text{Data}, \text{Prior}] > c$$

where

- $\rho_0$  is an effect threshold
- $c$  is probability threshold (often chosen to control type I error)
- ‘Data’ is the observed trial data
- ‘Prior’ is specified prior distribution on the treatment effect.

# Bayesian decision-making: example

- Assume that the planned sample size is  $N = 13$  patients;
- The null response rate is  $p_0 = 0.15$
- Clinically interesting response rate is  $p_1 = 0.45$

The type I error is defined as

$$\text{Prob}\{\text{Prob}[\theta \geq p_0 \mid \text{Data}, \text{Prior}] > c \mid \theta = p_0\}$$

The power is defined as

$$\text{Prob}\{\text{Prob}[\theta \geq p_0 \mid \text{Data}, \text{Prior}] > c \mid \theta = p_1\}$$



## Scenarios to consider

For a study with 5 baskets:

	$\rho_1$	$\rho_2$	$\rho_3$	$\rho_4$	$\rho_5$
Scenario 1	0.15	0.15	0.15	0.15	0.15
Scenario 2	0.45	0.15	0.15	0.15	0.15
Scenario 3	0.45	0.45	0.15	0.15	0.15
Scenario 4	0.45	0.45	0.45	0.15	0.15
Scenario 5	0.45	0.45	0.45	0.45	0.15
Scenario 6	0.45	0.45	0.45	0.45	0.45

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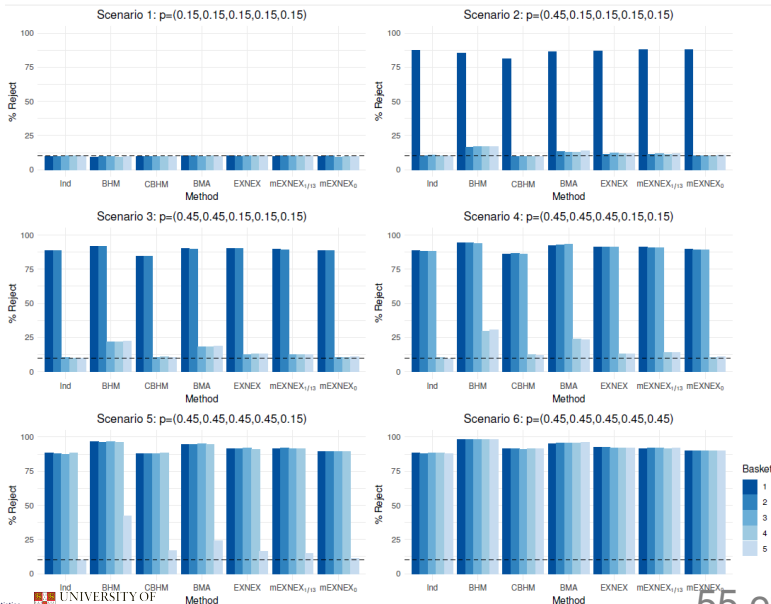
Equal sample sizes?

Moderate treatment effect?

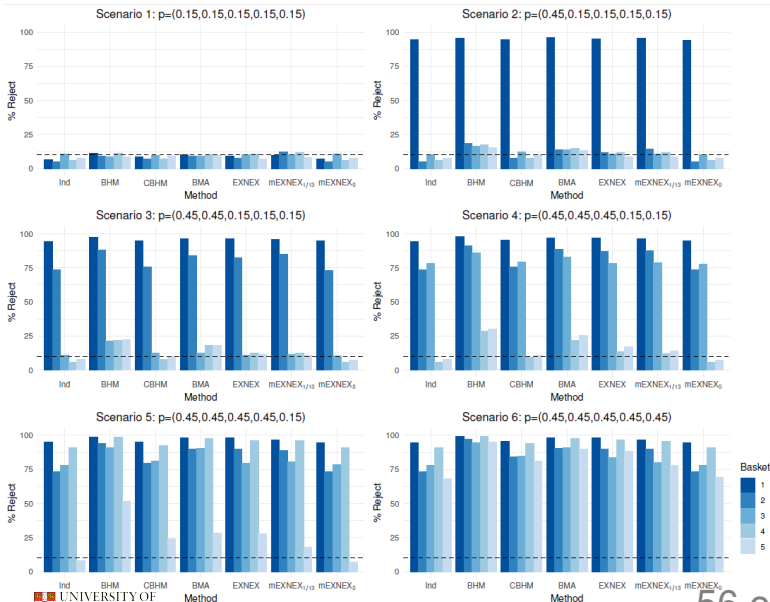
# Null configuration

- What would be the null hypothesis in a basket trial setting?
- With  $K$  baskets, one can consider the configuration  $p_1 = p_2 = \dots = p_k = p_0$  where  $p_0$  is a null response rate for the calibration of the probability threshold  $c$ .
- This assumes that all baskets are homogeneous. What if some of the baskets has interesting treatment effect and some null?
- Were one to calibrate under the case of the maximum type I error, there will be no more advantages of using borrowing (Kopp-Schneider et al. 2020)

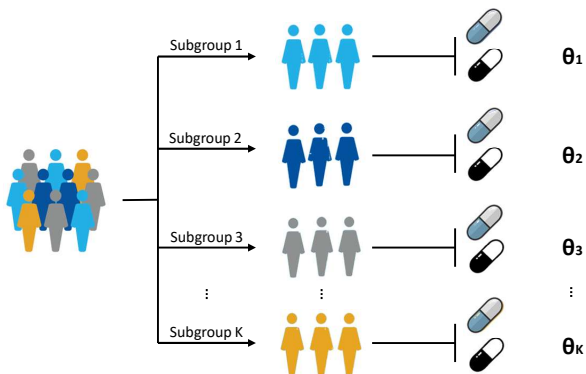
# Results: Planned sample size Daniells et al. (2023)



# Results: Realised sample size (as in practical)



# Randomised controlled basket trials



In a single-arm setting:

$$Y_i \mid p_i, n_i \sim \text{Binomial}(p_i, n_i)$$
$$\text{logit}(p_i) = \theta_i$$

In a randomised controlled setting:

$$Y_{jk} \mid p_{ji}, n_{ji} \sim \text{Binomial}(p_{ji}, n_{ji}), j = E, C,$$
$$\text{logit}(p_{Ci}) = \alpha_i$$
$$\text{logit}(p_{Ei}) = \alpha_i + \theta_i$$

where an uninformative prior is placed on  $\alpha_j$ , while  $\theta_i \mid \mu, \tau \sim N(\mu, \tau^2)$  to implement borrowing.



To observe  $n$  patients.

Patients are randomised to receive the new treatment (w.p.  $R$ ) or the control (w.p.  $1 - R$ ).

Let  $\bar{X}_i$  be the mean response per treatment group  $i = E, C$ , so that

$$\bar{X}_E - \bar{X}_C \sim N\left(\theta, \frac{\sigma^2}{nR(1-R)}\right), \quad \text{with } \theta = \mu_E - \mu_C.$$

Assume known  $\sigma^2$ .

Specify a prior on the difference in means,  $\theta \sim N(m, s^2)$ .

Establish **efficacy** if  $\Pr(\theta \geq 0 \mid \text{data, prior}) \geq \eta$ , or  
**futility** if  $\Pr(\theta < \delta \mid \text{data, prior}) \geq \zeta$ .

Under the assumption of no borrowing, the sample size required is

$$n \geq \frac{\sigma^2}{R(1-R)} \left[ \left( \frac{z_\eta + z_\zeta}{\delta} \right)^2 - \frac{1}{s^2} \right].$$

# Sample size formula under commensurate prior

Zheng et al. (2023)

## Recall

$$\theta_i \mid \theta_q, \nu_{qi} \sim N(\theta_q, \nu_{qi}^{-1}), \forall i = 1, \dots, k$$

$$\nu_{qi} \sim w_{qi} \text{Gamma}(a_1, b_1) + (1 - w_{qi}) \text{Gamma}(a_2, b_2), \text{ with } q \neq i$$

- Setting  $w_{qi} \rightarrow 0$  means strong borrowing

For basket-specific sample sizes  $n_1, \dots, n_k$  satisfy

$$\frac{R_k(1 - R_k)n_k}{\sigma_k^2} \left[ \sum_q \alpha_{qk}^2 \left( \left( \frac{1}{s_{0q}^2} + \frac{R_q(1 - R_q)n_q}{\sigma_q^2} \right)^{-1} + \frac{w_{qk}b_1}{a_1 - 1} + \frac{(1 - w_{qk})b_2}{a_2 - 1} \right) \right]^{-1} \geq \left( \frac{z_\eta + z_\zeta}{\delta} \right)^2 \forall k \neq q$$

- With  $0 \leq w_{qk} \leq 1$ , sample size saving can be expected

Apply Newton's method for systems of nonlinear equations to find  $n_1, \dots, n_k$ , simultaneously

## If you are a little rusty on Newton's method...

Sample size formulae for cases of no borrowing and commensurate borrowing are implemented in an online R Shiny App

<http://shiny.mrc-bsu.cam.ac.uk/apps/BasketTrials/>.

# R Shiny App: Sample Size Formula

# of Subtrials

$\delta$ : effect size

$\eta$ : efficacy threshold

$\zeta$ : futility threshold

outcome variances

$\sigma^2_1$

$\sigma^2_2$

$\sigma^2_3$

$\sigma^2_4$

$\sigma^2_5$

$a_1$

$b_1$

$a_2$

$b_2$

outcome means

$\mu_1$

$\mu_2$

$\mu_3$

$\mu_4$

$\mu_5$

Enable borrowing:



Level of Borrowing:

Moderate  Strong  Pairwise

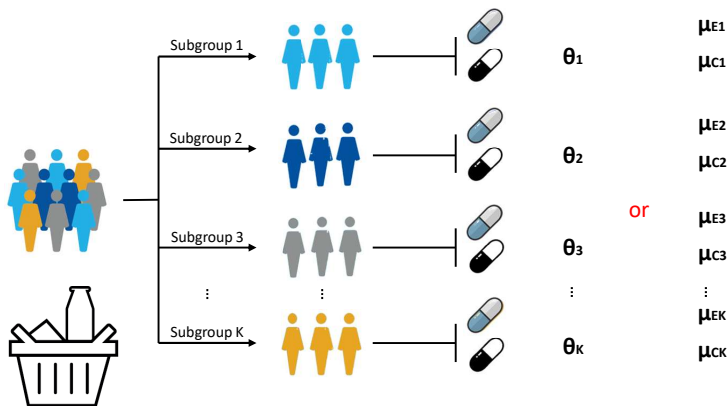
Use custom allocation ratios:



Results:






k	$\sigma_k$	n	$n_E$	$n_C$
1	10	136	68	68
2	10	78	39	39

# An alternative strategy for borrowing Ouma et al. (2022)



- Strong justification (such as a common genetic make-up or disease trait)  $\Rightarrow$  borrowing
- In data analysis, borrowing leads to higher statistical power than no borrowing
- If accounting for this formally in the design stage, borrowing means a smaller sample size to reach the same desired level of power/decision accuracy
- Inflation of type I error rates under BHM (Freidlin and Korn, 2013) and other borrowing methods

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