

# Advantages of using the Bayesian Framework for Modelling the relationship between Volume, Freezing Rate, Supercooling and Aggregation in Vaccines

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Ulcerative colitis at 40x magnification

# Introduction

Can aggregation be prevented by freezing larger volumes and using fast freezing ranges?



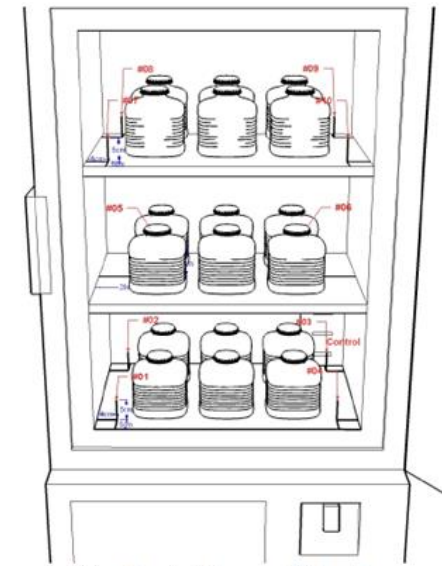
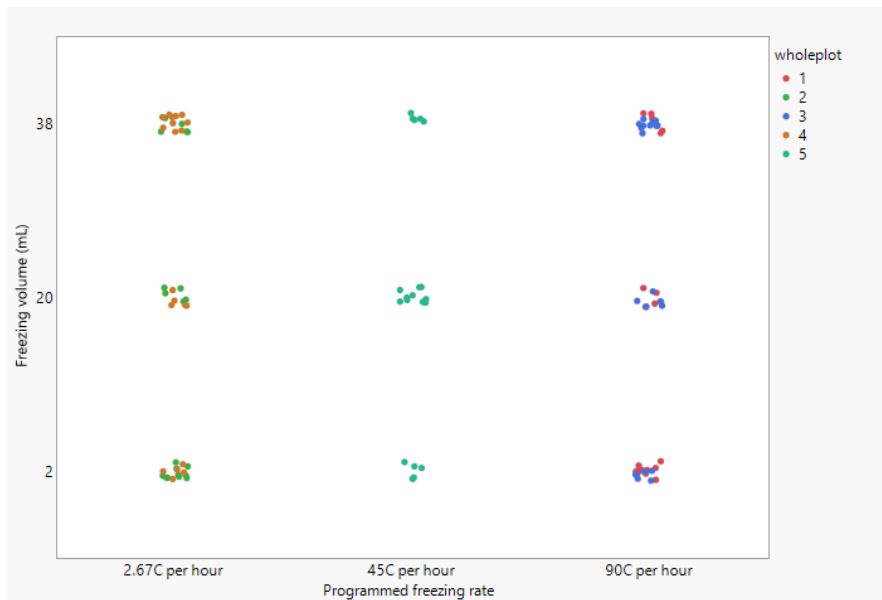
# Design of experiments

Split plot design with 5 whole plots with 20 experimental runs per whole plot

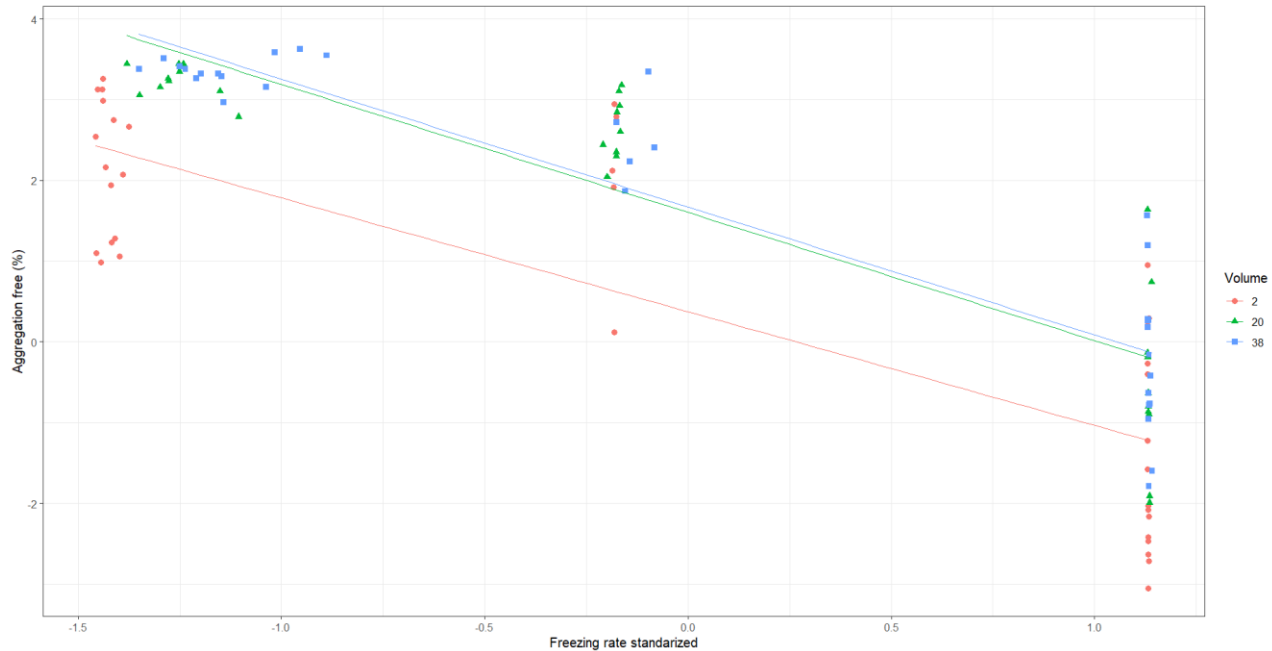
Factors included: Freezing Volume and freezing rate

Model: Response Surface Model (main and quadratic effects and second order interactions)

The experiment was performed with 3 different batches

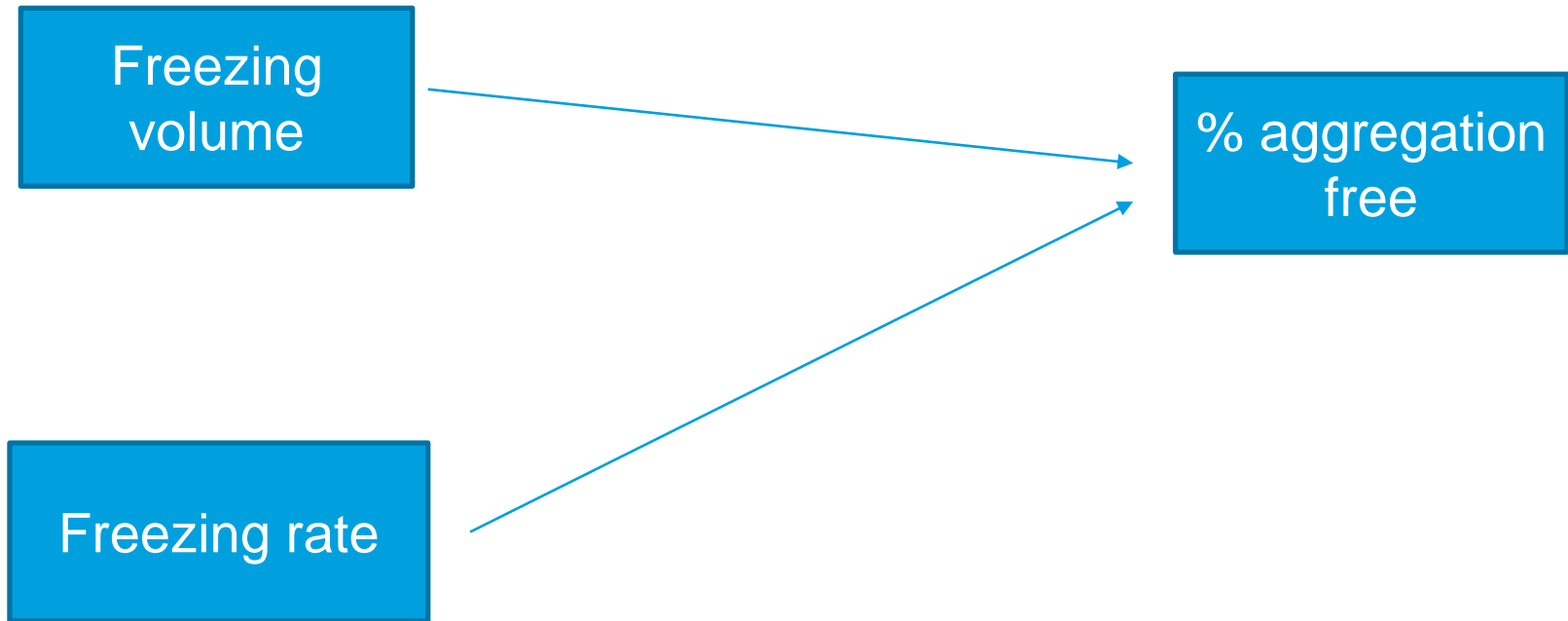


# Results aggregation free (%)



- Higher volume ➡ lower % aggregation
- Higher freezing rate ➡ lower % aggregation
- Heterogeneous variance

# Results aggregation



- The proposed design studies the direct influence of freezing volume and freezing rate on %aggregation free.

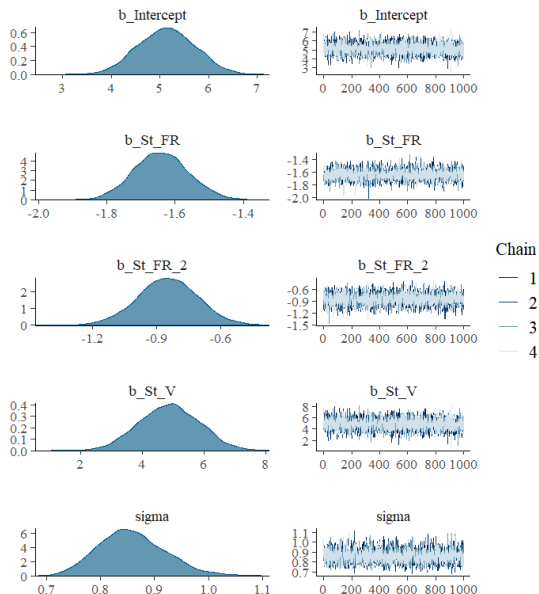
# Results aggregation

A Student-t prior with 3 degrees of freedom was used for the intercept and a half-student-t prior with 3 degrees of freedom was used for the standard deviation.

Whole plot random effect was excluded because the between whole plots variation was very small compared to the within plot variation.

$$\text{logit}(y_i) \sim \text{Normal}(\mu_i, \sigma)$$

$$\mu_i = \alpha_1 + \beta FR + \varphi FR^2 + \gamma V$$



Family: gaussian  
 Links: mu = identity; sigma = identity  
 Formula: logit\_ag ~ St\_FR + St\_FR\_2 + St\_V  
 Data: data11 (Number of observations: 96)  
 Draws: 4 chains, each with iter = 2000; warmup = 1000; thin = 1;  
 total post-warmup draws = 4000

## Population-Level Effects:

	Estimate	Est.Error	l-95% CI	u-95% CI	Rhat	Bulk_ESS	Tail_ESS
Intercept	5.13	0.60	3.99	6.30	1.00	5008	3392
St_FR	-1.64	0.08	-1.80	-1.48	1.00	4032	2648
St_FR_2	-0.85	0.14	-1.13	-0.57	1.00	4018	3163
St_V	4.87	0.98	3.00	6.80	1.00	4711	3327

## Family Specific Parameters:

	Estimate	Est.Error	l-95% CI	u-95% CI	Rhat	Bulk_ESS	Tail_ESS
sigma	0.86	0.06	0.75	0.99	1.00	3618	2943

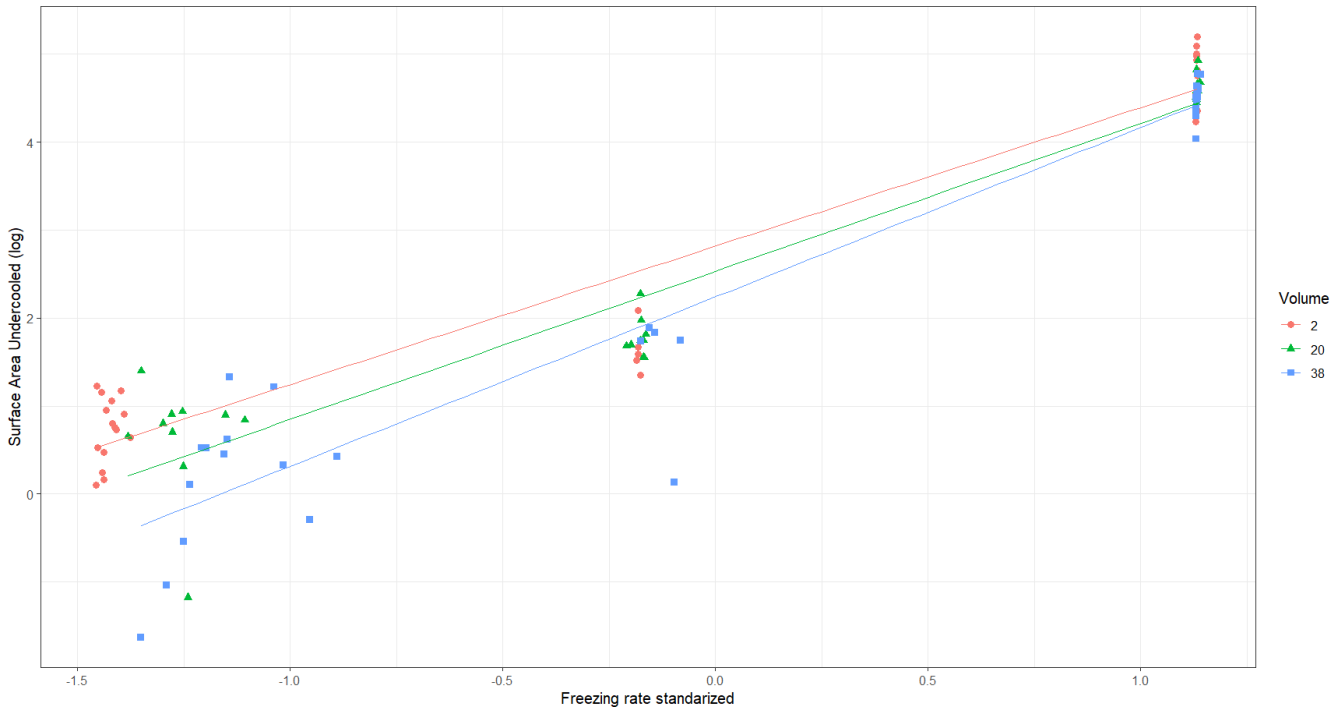
# Supercooling

Measurements performed during the experiments identify a new covariate that could be measured, but not directly manipulated. This covariate is called supercooling

Definition: It is the phenomenon where a substance remains in a liquid state at a temperature below its normal freezing point.

Measurements: surface area undercooled.

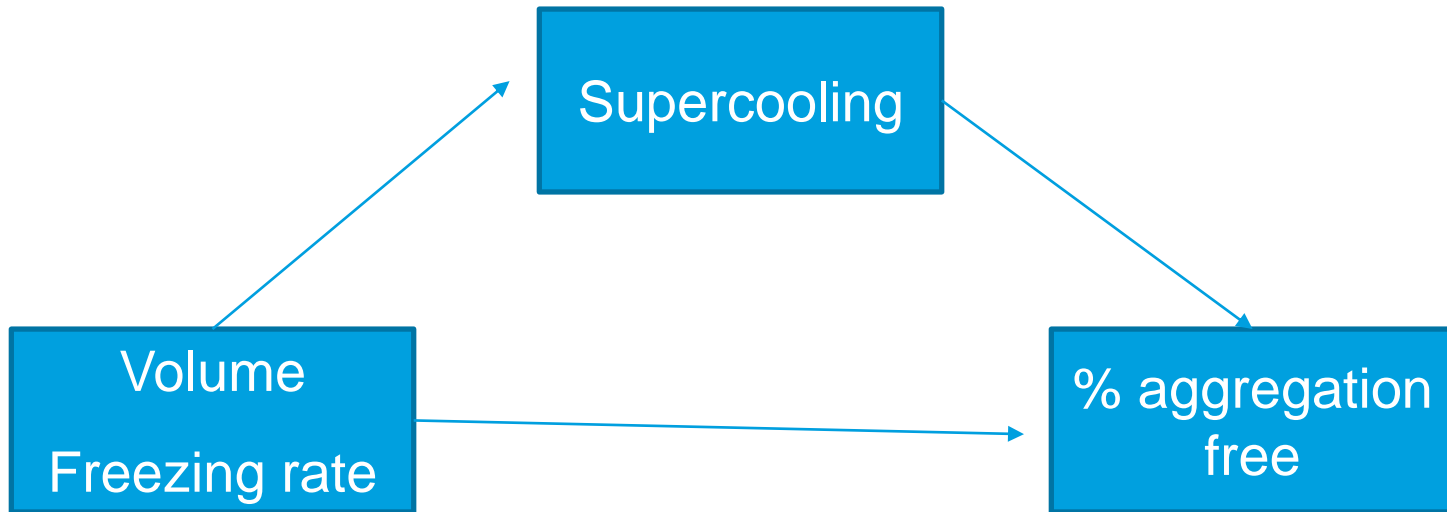
# Supercooling



- Lower volume → Larger area undercooled
- Slower freezing rate → Larger area undercooled



# Results aggregation free (%)



- Rather than a direct relationship between the experimental factors and %aggregation, volume and Freezing rage produce supercooling, which produces %aggregation.

# Statistical Mediation

It is interesting to study the relationship between the covariate and the factors, and between the covariate and the outcome of the experiment because the covariate was a key to understand the kinetic aspects of the freezing of the product

- Mediation is a three variables system in which an independent variable causes a mediating variable, which, in turn, causes a dependent variable (Baron & Kenny, 1986; MacKinnon, 2008).
- The aim of mediation analysis is to determine whether the relation between the independent variable and the dependent variable is due, wholly or in part, to the mediating variable.

$$Y = \alpha_1 + \beta FR + \gamma V + e_1$$

$$M = \alpha_2 + \rho FR + \varphi V + e_2$$

$$Y = \alpha_3 + \delta M + \beta' FR + \gamma' V + e_3$$

# Mediation analysis using Bayesian Regression model

- WHY?

- Probabilistic interpretation of the parameters and the conditional relationships between the variables.
- Predictive distributions of aggregation outside the study range.
- Possibility of updating the model with new information collected for larger volumes. ...

# Results

```
library(brms)
```

```
f1 <- bf(log_sup ~ St_FR + St_FR_2 + St_V)
```

```
f2 <- bf(logit_ag ~ St_FR + St_FR_2 + St_V + log_sup)
```

```
m12 <- brm(f1 + f2 + set_rescor(FALSE), data = data11,  
cores = 4)
```

```
mediation(m2)
```

- Direct effect: median value of posterior samples from treatment of the outcome model
- Mediator effect: median value of posterior samples from mediator of the outcome model.
- Indirect effect: median value of the multiplication of the posterior samples from mediator of the outcome model and the posterior samples from treatment of the mediation model.
- Total effect: median value of sums of posterior samples used for the direct and indirect effect.
- The proportion mediated is the indirect effect divided by the total effect.

# Results

Treatment: St\_FR  
Mediator : log\_sup  
Response : logitag

Effect	Estimate	95% ETI
Direct Effect (ADE)	-0.460	[-1.116, 0.205]
Indirect Effect (ACME)	-1.175	[-1.817, -0.528]
Mediator Effect	-0.658	[-1.015, -0.295]
Total Effect	-1.641	[-1.808, -1.471]

Proportion mediated: 71.62% [31.33%, 111.91%]

Treatment: St\_V  
Mediator : log\_sup  
Response : logitag

Effect	Estimate	95% ETI
Direct Effect (ADE)	3.541	[ 1.595, 5.362]
Indirect Effect (ACME)	1.274	[ 0.471, 2.410]
Mediator Effect	-0.653	[-1.008, -0.330]
Total Effect	4.850	[ 2.953, 6.781]

Proportion mediated: 26.27% [5.14%, 47.40%]

# Results

$$\text{logit}(y_i) \sim \text{Normal}(\mu_i, \sigma)$$

$$Y_i = \alpha_1 + \beta FR + \varphi FR^2 + \gamma V + \delta M + \theta MV$$

```
library(brms)
```

```
M_1 <- brm(logit_ag ~ St_FR + St_FR_2 + St_V + log_sup +  
log_sup:St_V, prior=bprior, data=data11, cores=4 )
```

# Results

The relationship between freezing rate and aggregation is mediated by supercooling, and the effect of supercooling on aggregation is higher for larger volumes.

```

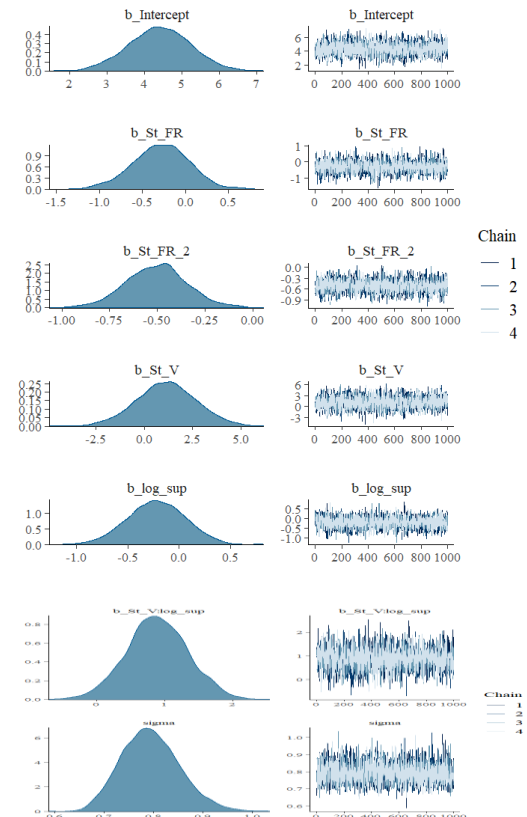
Family: gaussian
Links: mu = identity; sigma = identity
Formula: logit_ag ~ St_FR + St_FR_2 + St_V + log_sup + log_sup:St_V
Data: data11 (Number of observations: 96)
Draws: 4 chains, each with iter = 2000; warmup = 1000; thin = 1;
total post-warmup draws = 4000
  
```

## Population-Level Effects:

	Estimate	Est.Error	l-95% CI	u-95% CI	Rhat	Bulk_ESS	Tail_ESS
Intercept	4.412	0.829	2.776	6.014	1.001	2356	2644
St_FR	-0.284	0.336	-0.973	0.359	1.002	1886	1803
St_FR_2	-0.505	0.161	-0.826	-0.174	1.000	2335	2114
St_V	1.136	1.551	-1.946	4.184	1.002	2170	2435
log_sup	-0.221	0.280	-0.781	0.317	1.000	1990	2198
St_V:log_sup	0.890	0.450	0.016	1.771	1.001	1930	2093

## Family Specific Parameters:

	Estimate	Est.Error	l-95% CI	u-95% CI	Rhat	Bulk_ESS	Tail_ESS
sigma	0.794	0.059	0.688	0.918	1.000	2781	2622



# Validation runs for larger volumes

Programmed FR	Measured FR	Volume.(mL)	logit Aggr	Predicted Old Model	Q2.5	Q97.5	Predicted new model	Q2.5	Q97.5
90C per hour	-84	100	3.44	5.25	3.28	7.20	3.84	1.63	6.05
90C per hour	-70	100	3.41	5.34	3.36	7.24	4.17	1.94	6.31
90C per hour	-70	100	3.44	5.37	3.46	7.32	4.27	2.15	6.47
90C per hour	-56	100	3.41	5.20	3.24	7.15	4.60	2.27	6.86
90C per hour	-68	100	3.48	5.35	3.42	7.26	4.48	2.12	6.81
90C per hour	-62	250	3.62	9.77	6.51	12.82	5.30	0.97	9.64
90C per hour	-62	250	3.62	9.79	6.66	12.92	5.12	0.41	9.80
90C per hour	-64	250	3.66	9.79	6.63	12.90	5.23	0.66	9.71
90C per hour	-64	250	3.71	9.80	6.60	12.97	4.70	-0.67	10.06
90C per hour	-68	250	3.71	9.84	6.73	13.10	4.75	-0.44	10.03
90C per hour	-53	500	3.66	17.02	11.17	22.82	4.38	-7.54	16.27
90C per hour	-55	500	3.75	17.10	11.39	22.79	3.79	-8.75	16.33
90C per hour	-53	500	3.66	17.02	11.25	22.81	4.02	-8.30	16.21
90C per hour	-57	500	3.66	17.15	11.38	23.00	4.11	-8.19	16.35
90C per hour	-75	500	3.58	17.29	11.41	23.05	5.74	-4.42	15.86
90C per hour	-71	500	3.58	17.27	11.43	23.26	5.20	-5.79	15.94
90C per hour	-63	500	3.62	17.24	11.31	23.09	3.76	-8.88	16.42
90C per hour	-69	500	3.66	17.30	11.48	23.00	5.72	-4.76	15.93
90C per hour	-77	500	3.62	17.28	11.52	23.04	6.13	-3.49	15.71



# Summary

- The mediation analysis helped us to understand the relation between freezing rate, volume and aggregation and to build a model that could predict aggregation for larger volumes better than the original model.
- The relationship between freezing rate and percentage of aggregation free was mediated by supercooling and the relationship between supercooling and percentage of aggregation free changed across volumes.

# Thank you very much!

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